Chapter Six Review Key
RK – AA – U1C6

1. List all possible rational zeros of the function $f(x) = x^3 - x^2 - 9x + 9$. Do not find the zeros.

$$\frac{p}{q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1} = \frac{\pm 1}{\pm 1}, \frac{\pm 3}{\pm 1}, \frac{\pm 9}{\pm 1} = \pm 1, \pm 3, \pm 9$$

***Possible rational zeros*: ±1, ±3, ±9**

2. List all possible rational zeros of the function $f(x) = 4x^3 + 2x^2 + 16x + 8$. Do not find the zeros.

$$\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2, \pm 4} = \frac{\pm 1}{\pm 1}, \frac{\pm 2}{\pm 1}, \frac{\pm 4}{\pm 1}, \frac{\pm 8}{\pm 1}, \frac{\pm 1}{\pm 2}, \frac{\pm 2}{\pm 2}, \frac{\pm 4}{\pm 2}, \frac{\pm 8}{\pm 2}, \frac{\pm 1}{\pm 4}, \frac{\pm 2}{\pm 4}, \frac{\pm 4}{\pm 4}, \frac{\pm 8}{\pm 4} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}$$

***Possible rational zeros*:** $\mathbf{\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}}$

3. Solve the following equation, giving exact answers: $x^3 - 2x^2 - 9x + 18 = 0$.

$x^3 - 2x^2 - 9x + 18 = 0$	Original equation
$\underline{x^3 - 2x^2}\ \underline{- 9x + 18} = 0$	Grouping
$x^2(x - 2) - 9(x - 2) = 0$	Factoring by grouping
$(x^2 - 9)(x - 2) = 0$	Factoring
$(x + 3)(x - 3)(x - 2) = 0$	Factoring
$x + 3 = 0 \quad x - 3 = 0 \quad x - 2 = 0$	Zero Product Property
$x = -3 \quad x = 3 \quad x = 2$	Addition/Subtraction

***Solutions*: –3, 3, 2**

4. Solve the following equation, giving exact answers: $x^4 + x^2 = 2$.

$x^4 + x^2 = 2$	Original equation
$x^4 + x^2 - 2 = 0$	Subtraction
$(x^2 + 2)(x^2 - 1) = 0$	Factoring
$x^2 + 2 = 0 \quad x^2 - 1 = 0$	Zero Product Property
$x^2 = -2 \quad x^2 = 1$	Addition/Subtraction
$x = \pm\sqrt{-2} \quad x = \pm\sqrt{1}$	Square roots of each side
$x = \pm i\sqrt{2} \quad x = \pm 1$	Simplifying radicals

***Solutions*:** $\mathbf{\pm i\sqrt{2}, \pm 1}$

5. Solve the following equation, giving exact answers: $x^2 - 12x = -28$.

$x^2 - 12x = -28$	Original equation
$x^2 - 12x + \left(\frac{-12}{2}\right)^2 = -28 + \left(\frac{-12}{2}\right)^2$	Completing the square
$x^2 - 12x + 36 = -28 + 36$	Simplifying fractions and exponents
$x^2 - 12x + 36 = 8$	Addition
$(x - 6)^2 = 8$	Factoring

$x-6=\pm\sqrt{8}$	Square roots of each side
$x-6=\pm 2\sqrt{2}$	Simplifying radicals
$x=6\pm 2\sqrt{2}$	Addition

***Solutions*: $6\pm 2\sqrt{2}$**

6. Solve the following equation, giving exact answers: $(x-2)^2 + 64 = 72$.

$(x-2)^2 + 64 = 72$	Original equation
$(x-2)^2 = 8$	Subtraction
$x-2=\pm\sqrt{8}$	Square roots of each side
$x-2=\pm 2\sqrt{2}$	Simplifying radicals
$x=2\pm 2\sqrt{2}$	Addition

***Solutions*: $2\pm 2\sqrt{2}$**

7. Solve the following equation, giving exact answers: $4x^2 = 12x + 40$.

$4x^2 = 12x + 40$	Original equation
$4x^2 - 12x - 40 = 0$	Subtraction
$4(x^2 - 3x - 10) = 0$	Factoring out greatest common factor
$4(x-5)(x+2) = 0$	Factoring
$x-5=0 \quad x+2=0$	Zero Product Property
$x=5 \quad x=-2$	Addition/Subtraction

***Solutions*: 5, –2**

8. Solve the following equation, giving exact answers: $x^4 + x^3 + 2x^2 + 4x = 8$.

$x^4 + x^3 + 2x^2 + 4x = 8$	Original equation
$x^4 + x^3 + 2x^2 + 4x - 8 = 0$	Subtraction

Using the Rational Zeros Theorem, you would see that –2 and 1 are two of the "suspects" that can be zeros. Test this with synthetic division (as well as dividing down the polynomial to something solvable).

–2	1	1	2	4	–8
		–2	2	–8	8
	1	–1	4	–4	0

Since there is a remainder of zero, –2 is a solution. Using what remains and see if 1 works.

1	1	–1	4	–4
		1	0	4
	1	0	4	0

Since there is a remainder of zero, 1 is a solution. This also leave the remaining polynomial of $x^2 + 4 = 0$ ($x^2 + 0x + 4 = 0$).

$x^2 + 4 = 0$	Equation
$x^2 = -4$	Subtraction
$x=\pm\sqrt{-4}$	Square roots of each side

$x = \pm 2i$ Simplifying radicals

Solutions*: –2, 1, ±2*i

9. Write the following polynomial in standard form. Also classify it by number of terms and degree.
Polynomial: $(x^2 + 2x + 3) - (x^2 - 5)$

$(x^2 + 2x + 3) - (x^2 - 5)$	Original expression
$x^2 + 2x + 3 - x^2 + 5$	Distributing the negative
$x^2 - x^2 + 2x + 3 + 5$	Grouping like terms
$2x + 8$	Addition/Subtraction

***Standard form*: $2x + 8$**

***Name by degree*: Linear** (since degree is 1)

***Name by number of terms*: Binomial** (since there are only 2 terms)

10. Write the following polynomial in standard form. Also classify it by number of terms and degree.
Polynomial: $(6x^3 + 3x^2 - 5x - 1) - (7x^3 - 5x - 6)$

$(6x^3 + 3x^2 - 5x - 1) - (7x^3 - 5x - 6)$	Original expression
$6x^3 + 3x^2 - 5x - 1 - 7x^3 + 5x + 6$	Distributing the negative
$6x^3 - 7x^3 + 3x^2 - 5x + 5x - 1 + 6$	Grouping like terms
$-x^3 + 3x^2 + 5$	Addition/Subtraction

***Standard form*: $-x^3 + 3x^2 + 5$**

***Name by degree*: Cubic** (since degree is 3)

***Name by number of terms*: Trinomial** (since there are only 3 terms)

11. Write the following polynomial in standard form. Also classify it by number of terms and degree.
Polynomial: $(2x + 3) + (4x^2 - 10)$

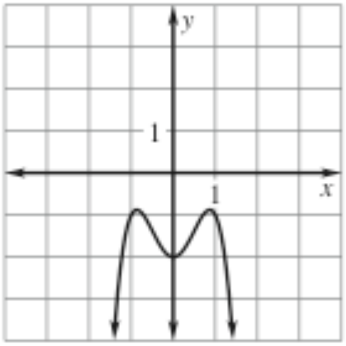
$(2x + 3) + (4x^2 - 10)$	Original expression
$2x + 3 + 4x^2 - 10$	Distributing the positive
$4x^2 + 2x + 3 - 10$	Grouping like terms
$4x^2 + 2x - 7$	Subtraction

***Standard form*: $4x^2 + 2x - 7$**

***Name by degree*: Quadratic** (since degree is 2)

***Name by number of terms*: Trinomial** (since there are only 3 terms)

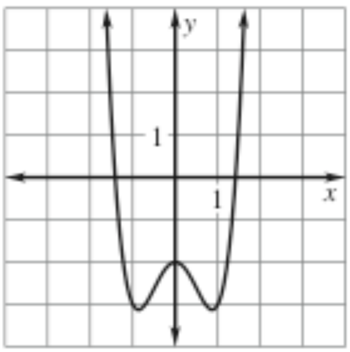
12. Use the graph below to approximate any relative minimums and maximums.



12. ***Max(s)*: (–1, –1); (1, –1)**

***Min(s)*: (0, –2)**

13. Use the graph below to approximate any relative minimums and maximums.



13. ***Max(s)*: (0, –2)**

***Min(s)*: (–1, –3); (1, –3)**

14. Divide ($x^4 + 9x^3 – 4x – 17$) ÷ ($x + 5$).

<u>BE SURE TO FILL IN THE MISSING COEFFICIENTS OF $x^4 + 9x^3 – 4x – 17$</u> (since this is the same as saying $x^4 + 9x^3 + 0x^2 – 4x – 17$). Realize you are dividing by –5 since $x + 5 = 0$, therefore you can use synthetic division

–5	1	9	0	–4	–17
		–5	–20	100	–480
	1	4	–20	96	–497

***Quotient*: $\boldsymbol{x^3 + 4x^2 – 20x + 96 – \frac{497}{x+5}}$**

15. Divide ($5x^4 + 14x^3 + 9x$) ÷ ($x^2 + 3x$).

<u>BE SURE TO FILL IN THE MISSING COEFFICIENTS OF $5x^4 + 14x^3 + 9x$ and $x^2 + 3x$</u> (since this is the same as saying $5x^4 + 14x^3 + 0x^2 + 9x + 0$ and $x^2 + 3x + 0$, respectively). Since the divisor is not a linear divisor, you have to use long division.

$$
\begin{array}{r|l}
 & \quad 5x^2 \; - \; x+3 \\
\hline
x^2+3x+0 & \quad 5x^4+14x^3+0x^2+9x+0 \\
 & -\left(5x^4+15x^3+0x^2\right) \\
 & \qquad -x^3+0x^2+9x \\
 & \qquad -\left(-x^3-3x^2+0x\right) \\
 & \qquad\qquad 3x^2+9x+0 \\
 & \qquad\qquad -\left(3x^2+9x+0\right) \\
 & \qquad\qquad\qquad 0
\end{array}
$$

***Quotient*: $5x^2 - x + 3$**

16. Three of the roots of a polynomial are 5, $-4i$, and $1+\sqrt{6}$. What are all of the ***roots*** of this polynomial? Explain.

***Roots*: 5, 4*i*, –4*i*, $1+\sqrt{6}$, and $1-\sqrt{6}$**

***Explanation*: Since complex conjugates exist together as roots (Imaginary Roots Theorem), if one root is –4*i*, then 4*i* must be another root. Since irrational conjugates exist together as roots (Irrational Root Theorem), if one root is $1+\sqrt{6}$, then $1-\sqrt{6}$ be another root.**

17. Two of the roots of a polynomial are $-\sqrt{3}$ and 7*i*. What are all of the ***factors*** of this polynomial? Explain.

***Factors*: (x – 7*i*), (x + 7*i*), $\left(x-\sqrt{3}\right)$, and $\left(x+\sqrt{3}\right)$**

***Explanation*: Since complex conjugates exist together as roots (Imaginary Roots Theorem), if one root is 7*i*, then –7*i* must be another root. As factors, these would be (x – 7*i*) and (x + 7*i*), respectively. Since irrational conjugates exist together as roots (Irrational Root Theorem), if one root is $-\sqrt{3}$, then $\sqrt{3}$ be another root. As factors, these would be $\left(x+\sqrt{3}\right)$ and $\left(x-\sqrt{3}\right)$, respectively.**

18. Write the following function in factored form $f(x) = 2x^5 - 12x^4 + 18x^3$.

$f(x) = 2x^5 - 12x^4 + 18x^3$	Original equation
$f(x) = 2x^3(x^2 - 6x + 9)$	Factoring out greatest common factor
$f(x) = 2x^3(x - 3)(x - 3)$	Factoring
$f(x) = 2x^3(x - 3)^2$	Definition of square

***Factored form*: $f(x) = 2x^3(x - 3)^2$**

19. Find the zeros and multiplicity of zeros of the function from #18: $f(x) = 2x^5 - 12x^4 + 18x^3$.

$f(x) = 2x^5 - 12x^4 + 18x^3$		Original equation
$f(x) = 2x^3(x - 3)^2$		Factored form of original equation
$2x^3 = 0$	$(x - 3)^2 = 0$	Zero Product Property
$x^3 = 0$		Division
$x = 0$	$x - 3 = 0$	Roots of each side
	$x = 3$	

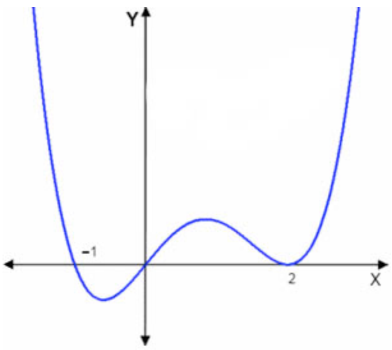
***Zeros*: 0 and 3** ***Multiplicities*: Multiplicity of 3 for 0 and multiplicity of 2 for 3**

20. Describe the end behavior of the function $f(x) = 2x^5 - 12x^4 + 18x^3$ by filling in the blanks at right.

As $x \to -\infty$, $f(x) \to$ <u>$-\infty$</u> **As $x \to +\infty$, $f(x) \to$ <u>$+\infty$</u>**

This can be seen by graphing since, as your *x* becomes more negative, your *y* becomes more negative. Similarly, as *x* gets more positive, *y* gets more positive.

21. Write a possible function in factored form for the graph shown below.



One of the zeros is at –1, meaning a factor is (x + 1) (since $x = -1$, $x + 1 = 0$). Since it goes right through the x-axis, it has an odd multiplicity, meaning an odd exponent (such as 1). Another of the zeros is at 0, meaning a factor of x (or (x + 0) since $x = 0$). Since it goes right through the x-axis, it has an odd multiplicity, meaning an odd exponent (such as 1). One of the zeros is at 2, meaning a factor is (x – 2) (since $x = 2$, $x - 2 = 0$). Since it bounces off the x-axis, it has an even multiplicity, meaning an even exponent (such as 2). Since the overall graph is positive in both directions, it is a positive polynomial (positive leading coefficient.

Polynomial: **$f(x) = (x + 1)^1 x(x - 2)^2$**
$f(x) = x(x + 1)(x - 2)^2$ (simplified factored form)

22. Determine if (x – 4) is a factor of the function $f(x) = x^4 - 3x^2 + 5x - 8$. How does this method shown if this or is not a factor?

Plugging 4 (since $x - 4 = 0$ is what you are testing) will let you know if a remainder exists.

$f(4) = (4)^4 - 3(4)^2 + 5(4) - 8 = 64 - 3(16) + 5(4) - 8 = 64 - 48 + 20 - 8 = 28$

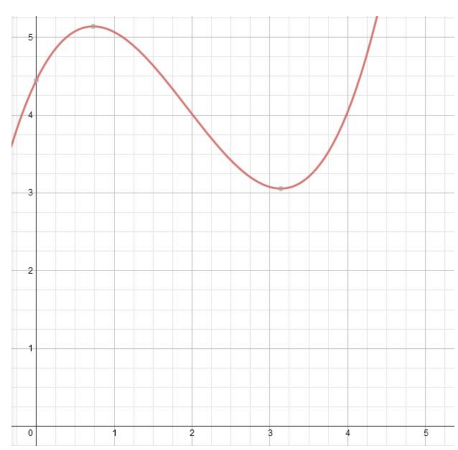
Dividing the function by $x - 4$ would leave a remainder of 28. Since it does not go into the function evenly, it is not a factor.

23. The average amount of tangarines (t in pounds) eaten per person each year in the United States from 2001 to 2006 can be modeled by $t = 0.298y^3 - 1.73y^2 + 2.05y + 4.45$ where y is the number of years since 2001. Using your graphing calculator:

a. Graph the function and identify the relative minimum and relative maximum where $0 \le y \le 4$.

To graph the function, press the "Y=" button on your calculator, clear all functions using the "CLEAR" button, and enter 0.298X^3 – 1.73X^2 + 20.05X + 4.45 (realizing X is really your *y* and Y is really your *t*). Press the "GRAPH" button. You can see a general graph. You can also Press the "2nd" button and then the "GRAPH" button to see a table of values.

a.



Relative minimum: **(3.14, 3.06)**

If you already have the graph in your calculator, press the "2nd" button and then the "TRACE" button to enter the CALC menu. Scroll down to "3:minimum" and press ENTER to set where to find the minimum. Set the "Left Bound" slightly left of where you see the minimum and press ENTER. Set the "Right Bound" slightly right of where you see the minimum and press ENTER. When the calculator asks "Guess?", press ENTER. The relative minimum is shown at about (3.14, 3.06).

Relative maximum: **(0.73, 5.14)**

If you already have the graph in your calculator, press the "2nd" button and then the "TRACE" button to enter the CALC menu. Scroll down to "4:maximum" and press ENTER to set where to find the maximum. Set the "Left Bound" slightly left of where you see the maximum and press ENTER. Set the "Right Bound" slightly right of where you see the maximum and press ENTER. When the calculator asks "Guess?", press ENTER. The relative minimum is shown at about (0.73, 5.14).

b. What is the real-life meaning of the relative minimum?

After about 2002 (0.73 years since 2001) to a little bit into 2004 (3.14 years since 2001), tangerine eating went down to 3.06 pounds per person.

c. What is the real-life meaning of the relative maximum?

From 2001 to about 2002 (0.73 years since 2001), tangerine eating went up to 5.14 pounds per person.

24. Use your graphing calculator to approximate the coordinates of the zeros, relative maximums, and relative minimum of the graph of the function listed below. Also identify the end behavior of the graph of the function.

Function: $f(x) = 0.25x^3 + 0.755x^2 - 1.06x - 1.17$

a. Zero(s) of the function: **(–3.81, 0), (–0.78, 0), (1.57, 0)**

To graph the function, press the "Y=" button on your calculator, clear all functions using the "CLEAR" button, and enter 0.25X^3 + 0.755X^2 – 1.06X – 1.17. Press the "GRAPH" button. You can see a general graph. Now, press the "2nd" button and then the "TRACE" button to enter the CALC menu. Scroll down to "2:zero" and press ENTER to set where to find the zero. Set the "Left Bound" slightly left of where you see a zero (where it crosses the *x*-axis) and press ENTER. Set the "Right Bound" slightly right of where you see the zero and press ENTER. When the calculator asks "Guess?", press ENTER. If you do this for both zeros, you will find zeros at (–3.81, 0), (–0.78, 0), and (1.57, 0).

b. Relative minimum: **(0.55, –1.48)**

If you already have the graph in your calculator, press the "2nd" button and then the "TRACE" button to enter the CALC menu. Scroll down to "3:minimum" and press ENTER to set where to find the minimum. Set the "Left Bound" slightly left of where you see the minimum and press ENTER. Set the "Right Bound" slightly right of where you see the minimum and press ENTER. When the calculator asks "Guess?", press ENTER. The relative minimum is shown at about (0.55, –1.48).

c. Relative maximum: **(–2.56, 2.30)**

If you already have the graph in your calculator, press the "2nd" button and then the "TRACE" button to enter the CALC menu. Scroll down to "4:maximum" and press ENTER to set where to find the maximum. Set the "Left Bound" slightly left of where you see the maximum and press ENTER. Set the "Right Bound" slightly right of where you see the maximum and press ENTER. When the calculator asks "Guess?", press ENTER. The relative minimum is shown at about (–2.56, 2.30).

d. End behavior: As $x \to -\infty$, $f(x) \to$ **$-\infty$** As $x \to +\infty$, $f(x) \to$ **$+\infty$**

This can be seen by graphing since, as your *x* becomes more negative, your *y* becomes more negative. Similarly, as *x* gets more positive, *y* gets more positive.