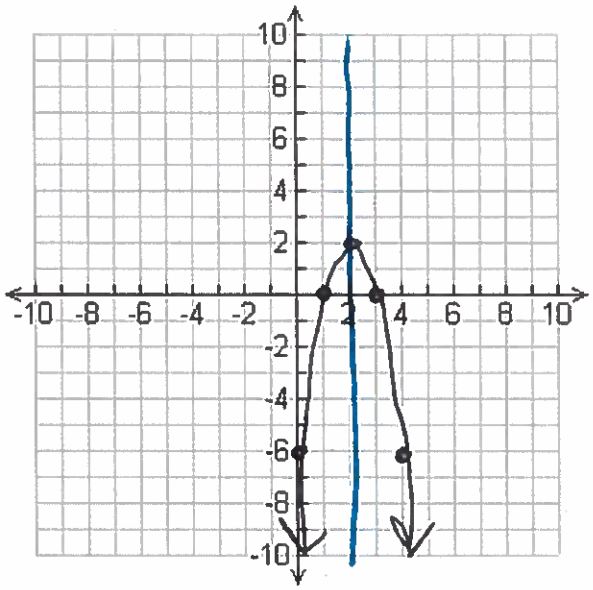
Chapter 5: Graph Quadratic Equations

1. Graph: $y = -2x^2 + 8x - 6$



Vertex: (2, 2)

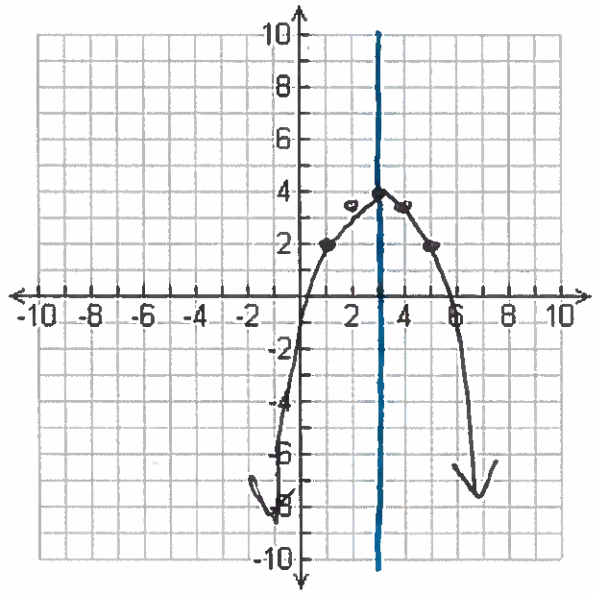
Axis of Symmetry: $x = 2$ $\quad \frac{-8}{2(-2)} = 2$

Y-intercept: -6

Is the vertex a max or min? max

X	Y
2	2
1	0
0	−6

2. Graph: $y = \frac{-1}{2}(x-3)^2 + 4$



Vertex: (3, 4)

Axis of Symmetry: $x = 3$

Y-intercept: $-.5$ $\quad -\frac{1}{2}(0-3)^2 + 4$

Is the vertex a max or min? max

Transformations:

Horizontal shift left or right? right 3

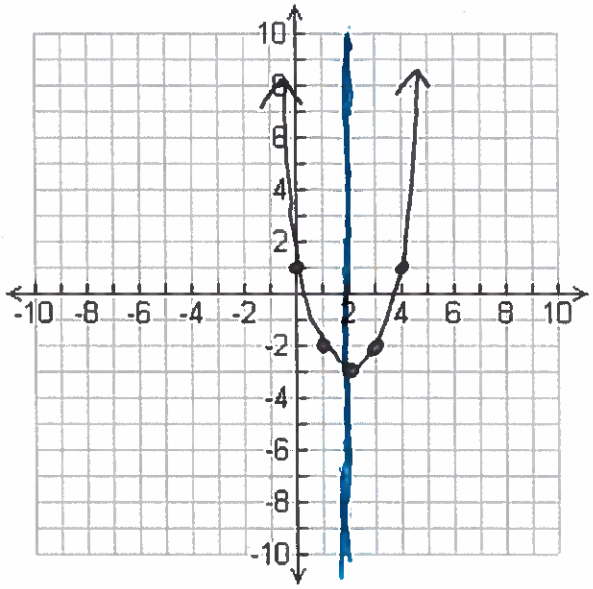
Vertical shift up or down? up 4

Vertical stretch or shrink? shrink

Reflection over the x-axis? (Yes) or No

X	Y
3	4
2	3.5
1	2

3. Graph: $y = x^2 - 4x + 1$



Vertex: $(2, -3)$ $\frac{4}{2(1)} = 2$

Axis of Symmetry: $x = 2$

Y-intercept: 1

Is the vertex a max or min? min

X	Y
2	−3
1	−2
0	1

Chapter 5: Simplify Quadratic and Complex Expressions

Factor the following quadratics. Remember to factor out a GCF if there is one.

1. $x^2 + 9x + 20$

$(x+5)(x+4)$

2. $x^2 - 49$

$(x-7)(x+7)$

3. $x^2 - 11x + 24$

$(x-8)(x-3)$

4. $x^2 + 2x - 99$

$(x+11)(x-9)$

5. $2x^2 - 72$

$2(x^2-36)$

$2(x+6)(x-6)$

6. $3x^2 + 51x + 90$

$3(x^2+17x+30)$

$3(x+15)(x+2)$

7. $4x^2 - 81$

$(2x-9)(2x+9)$

8. $2x^2 - 5x - 12$

$(2x+3)(x-4)$

9. $5x^2 + 16x + 3$

$(5x+1)(x+3)$

10. $12x^2 - 26x + 10$

$2(6x^2-13x+5)$

$2(3x-5)(2x-1)$

Write the expression as a complex number in standard form. (remember $i^2 = -1$)

11. $(6 - 5i) + (2 + 11i)$

$8 + 6i$

12. $(3 + i) - (7 - 6i)$

$3 + i - 7 + 6i$

$-4 + 7i$

13. $(2 - 7i)(-5 + 3i)$

$-10 + 6i + 35i - 21i^2$

$-10 + 41i + 21$

$11 + 41i$

14. $(4 - 11i)^2$

$(4-11i)(4-11i)$

$16 - 44i - 44i + 121i^2$

$16 - 88i - 121$

$-105 - 88i$

Chapter 5: Solve and Apply Quadratic Equations

Solve the quadratic equation by using any method. Justify your answer.

1. $n^2 + 8 = 80$

$n^2 + 8 = 80$

$-8 \quad -8$

$n^2 = 72$

$\sqrt{n^2} = \sqrt{72}$ $\quad \sqrt{36}\cdot\sqrt{2}$

$\boxed{n = \pm 6\sqrt{2}}$

2. $x^2 = 2x + 48$

$x^2 - 2x - 48 = 0$

$(x-8)(x+6) = 0$

$\boxed{x = 8}$ $\boxed{x = -6}$

3. $4x^2 - 1 = -8x$

$4x^2 + 8x - 1 = 0$

$$\frac{-8 \pm \sqrt{(8)^2 - 4(4)(-1)}}{2(4)}$$

$$\frac{-8 \pm \sqrt{80}}{8} \quad \sqrt{16}\cdot\sqrt{5} \quad = \quad \frac{-8 \pm 4\sqrt{5}}{8}$$

$$\boxed{\frac{-2 \pm \sqrt{5}}{2}}$$

4. $3r^2 - 16r - 7 = 5$

$3r^2 - 16r - 12 = 0$

$(3r+2)(r-6) = 0$

$3r + 2 = 0$ $\quad -2 \quad -2$

$\frac{3r}{3} = \frac{-2}{3}$

$\boxed{r = -2/3}$

$r - 6 = 0$ $\quad +6 \quad +6$

$\boxed{r = 6}$

5. $x^2 - 11x + 19 = -5$

$x^2 - 11x + 24 = 0$

$(x-8)(x-3) = 0$

$\boxed{x = 8}$ $\boxed{x = 3}$

6. $x^2 = -x + 1$

$x^2 + x - 1 = 0$

$$\frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$\boxed{\frac{-1 \pm \sqrt{5}}{2}}$$

7.

Firefighting Smoke jumpers are in free fall from the time they jump out of a plane until they open their parachutes. The function $y = -16t^2 + 1600$ models a jumper's height y in feet at t seconds for a jump from 1600 ft. How long is a jumper in free fall if the parachute opens at 1000 ft?

$1000 = -16t^2 + 1600$

$-1600 \quad\quad -1600$

$\frac{-600}{-16} = \frac{-16t^2}{-16}$

$37.5 = t^2$

$\sqrt{37.5} = \sqrt{t^2}$

$6.1 \approx t$ sec

8. A water balloon is catapulted into the air so that its height h, in metres, after t seconds is $h = -4.9t^2 + 27t + 2.4$

If graphing use the following window:

How long does it take for the water balloon to reach its maximum height?

2.76 sec

What is the maximum height of the water balloon?

39.59 m

9. A model rocket is launched from the roof of a building. Its flight path is modeled by $h = -5t^2 + 30t + 10$ where h is the height of the rocket above the ground in metres and t is the time after the launch in seconds.

If graphing use the following window:

How long does it take for the model rocket to reach its maximum height?

3 sec

What is the maximum height of the model rocket?

55 m

Chapter 6: Characteristics of Polynomial Functions

1. Write a polynomial function that has zeros 5, -2, and 0 in standard form. Classify the polynomial by degree and number of terms.

Factored Form: $y = x(x-5)(x+2)$

Standard Form: $y = x^3 - 3x^2 - 10x$

Name by Degree: cubic

Name by Number of Terms: trinomial

$x(x-5)(x+2)$

$(x^2-5x)(x+2)$

$x^3+2x^2-5x^2-10x$

x^3-3x^2-10x

2. Write a polynomial function that has zeros 0 with a multiplicity of 2, -3, and 4. Classify the polynomial by degree and number of terms.

Factored Form: $y = x^2(x+3)(x-4)$

Standard Form: $y = x^4 - x^3 - 12x^2$

Name by Degree: quartic

Name by Number of Terms: trinomial

$x^2(x+3)(x-4)$

$(x^3+3x^2)(x-4)$

$x^4-4x^3+3x^3-12x^2$

$x^4-x^3-12x^2$

3. A polynomial of least degree with rational coefficients has the given roots of 2, -6, and $2+\sqrt{3}$. Find all other roots.

$2-\sqrt{3}$.

4. A polynomial of least degree with rational coefficients has the given roots of 2, $1-\sqrt{11}$, and $1+2i$. Find all other roots.

$1+\sqrt{11}$ and $1-2i$

5. Divide $(x^3 + 2x^2 - x + 4)$ by $(x - 3)$

3	1	2	-1	4
	↓	3	15	42
	1	5	14	46

$x^2+5x+14$ R 46

6. Divide $(x^5 - 3x^3 + 2x - 6)$ by $(x+2)$

-2	1	0	-3	0	2	-6
	↓	-2	4	-2	4	-12
	1	-2	1	-2	6	-18

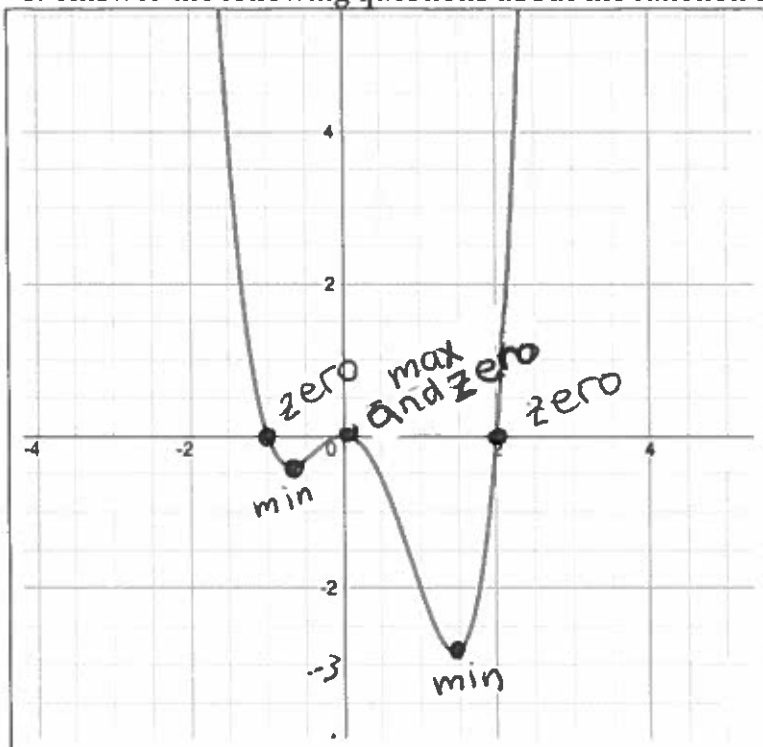
$x^4-2x^3+x^2-2x+6$ R -18

7. Divide $(x^4 - 3x^2 - 2)$ by $(x - 2)$

2	1	0	-3	0	-2
	↓	2	4	2	4
	1	2	1	2	2

x^3+2x^2+x+2 R 2

8. Answer the following questions about the function below.



Approximate the max(s)

$(0, 0)$

Approximate the min(s)

$(-.6, -.4)$ and $(1.4, -2.8)$

What is the end behavior?

$f(x) \to -\infty$ as $x \to \infty$
$f(x) \to +\infty$ as $x \to \infty$

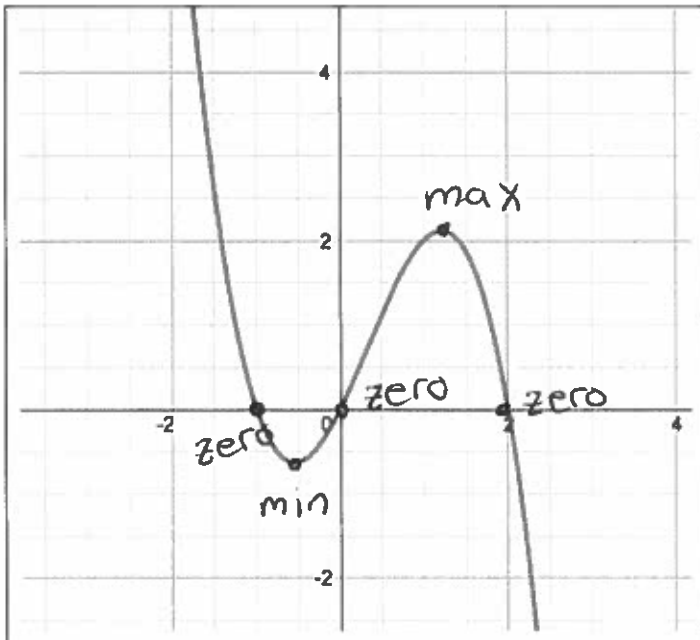
What are the zeros? (include multiplicity for multiple zeros)

-1, 0 mult of 2, 2

What is the equation of the polynomial in factored form?

$y = (x+1)(x^2)(x-2)$

9.



Approximate the max(s)

$(1.3, 2.2)$

Approximate the min(s)

$(-.6, -.6)$

What is the end behavior?

$f(x) \to -\infty$ as $x \to \infty$
$f(x) \to +\infty$ as $x \to -\infty$

What are the zeros? (include multiplicity for multiple zeros)

$-1, 0, 2$

What is the equation of the polynomial in factored form?

$y = (x+1)(x)(x-2)$

Chapter 6: Solve Polynomial Equations

1. List the possible rational zeros of $f(x) = 3x^4 - 2x^2 + 5x - 15$

$$\frac{\text{factors } 15}{\text{factors } 3} = \frac{1, 3, 5, 15}{1, 3} = \pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{3}, \pm \frac{5}{3}$$

2. List the possible rational zeros of $f(x) = 5x^3 - 4x^2 + 24$

$$\frac{\text{factors } 24}{\text{factors } 5} = \frac{1, 2, 12, 3, 8, 4, 6, 24}{1, 5} = \pm 1, \pm 2, \pm 12, \pm 3, \pm 8, \pm 4, \pm 6, \pm 24, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{12}{5}, \pm \frac{3}{5}, \pm \frac{8}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{24}{5}$$

3. Determine the zeros and the multiplicity of any multiple zeros.
$x^5 - 2x^4 + 8x^3$

$$x^3(x^2 - 2x + 8) = x^3(x-4)(x+2)$$

0 mult of 3, 4, and -2

4. Determine the zeros and the multiplicity of any multiple zeros.
$x^4 + 10x^3 + 25x^2$

$$x^2(x^2 + 10x + 25) = x^2(x+5)(x+5)$$

0 mult of 2, -5, and -5

Solve each equation:

5. $x^3 + 6x^2 + x + 6 = 0$

0 mult of 2, -5 mult of 2

-6 | 1 6 1 6
↓ -6 0 -6
1 0 1 0

$$x^2 + 1 = 0$$
$$-1 \quad -1$$
$$x^2 = -1$$

$$\sqrt{x^2} = \sqrt{-1}$$

$x = \pm i$

$x = -6$

6. $x^4 = x^2 + 20$

$$x^4 - x^2 - 20 = 0$$
$$(x^2 - 5)(x^2 + 4) = 0$$

$$x^2 - 5 = 0$$
$$+5 \quad +5$$
$$x^2 = 5$$
$$\sqrt{x^2} = \sqrt{5}$$
$\pm\sqrt{5}$

$$x^2 + 4 = 0$$
$$-4 \quad -4$$
$$x^2 = -4$$
$$\sqrt{x^2} = \sqrt{-4}$$
$\pm 2i$

7. $x^3 - 2x^2 - 5x + 6 = 0$

-2 | 1 −2 −5 6
↓ −2 8 −6
1 −4 3 0

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3$ $x = 1$

$x = -2$

8. $2x^4 - 14x^2 - 36 = 0$

$2(x^4 - 7x^2 - 18) = 0$

$2(x^2+2)(x^2-9) = 0$

$x^2 + 2 = 0$
$-2 \quad -2$
$x^2 = -2$
$\sqrt{x^2} = \sqrt{-2}$
$\pm i\sqrt{2}$

$x^2 - 9 = 0$
$+9 \quad +9$
$x^2 = 9$
$\sqrt{x^2} = \sqrt{9}$
± 3

9. $x^3 - 5x^2 + 17x - 13 = 0$

1 | 1 −5 17 −13
↓ 1 −4 13
1 −4 13 0

$x^2 - 4x + 13 = 0$

$$\frac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$\frac{4 \pm \sqrt{-36}}{2}$$

$$\frac{4 \pm 6i}{2} = 2 \pm 3i$$

$x = 1$