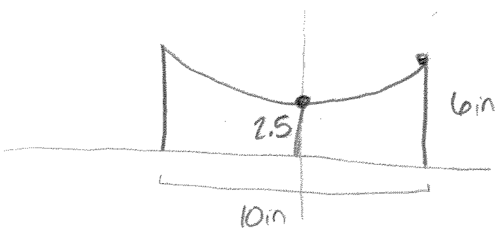
Review Boardwork

1. Two 6 inch sticks that are 10 inches apart have a chain suspended between them. The center of the chain is 2.5 inches above the ground. Find the equation of the catenary curve $y = b + a\cosh\left(\frac{x}{a}\right)$ that represents the chain.



$\overset{x\ y}{(5,6)}$ and $\overset{x\ \ y}{(0,2.5)}$

$2.5 = b + a\underbrace{\cosh\left(\frac{0}{a}\right)}$

$2.5 = b + a$

$b = 2.5 - a$

$6 = b + a\cosh\left(\frac{5}{a}\right)$

$6 = 2.5 - a + a\cosh\left(\frac{5}{a}\right)$

$0 = -3.5 - a + a\cosh\left(\frac{5}{a}\right)$

$a \approx 4.049$

$b = 2.5 - 4.049 = -1.549$

$\boxed{y = -1.549 + 4.049\cosh\left(\frac{x}{4.049}\right)}$

2. Find $\frac{d}{dx}f^{-1}(6)$ when $f(x) = \sqrt{x} + x$

$6 = \sqrt{x} + x$

$x = 4$

$a = 4$

$f'(x) = \frac{1}{2}(4)^{-\frac{1}{2}} + 1 = 1.25$

$\frac{d}{dx}f^{-1}(6) = \frac{1}{f'(4)}$

$= \frac{1}{1.25}$

or

$\frac{4}{5}$

3. Find the linearization function for $f(x) = \ln x$ at $x = 1$. Then approximate $x = 1.1$ and find the error.

$f(1) = \ln(1) = 0 \quad (1,0)$

$f'(x) = \frac{1}{x} \quad f'(1) = \frac{1}{1} = 1 =$ slope

$y - 0 = 1(x - 1)$

$\boxed{y = x - 1}$

$y = 1.1 - 1 = .1$ approx

$f(1.1) = .095$

error: .005

(4) $x^2 + 6xy - 4y^3 = 5$

a) Find $\frac{dy}{dx}$.

b) What is the equation of the tangent line at (1,0)

$2x + 6y + 6xy' - 12y^2y' = 0$

$6xy' - 12y^2y' = -2x - 6y$

$y'(6x - 12y^2) = -2x - 6y$

$$\boxed{y' = \frac{-2x-6y}{6x-12y^2}}$$

$$y' = \frac{-2(1)-6(0)}{6(1)-12(0)^2} = \frac{-1}{3}$$

$y - 0 = \frac{-1}{3}(x-1)$

$$\boxed{y = \frac{-1}{3}x + \frac{1}{3}}$$

(5) Find each derivative.

a) $f(x) = \cosh(\ln x)$

a) $f'(x) = \sinh(\ln x) \cdot \frac{1}{x}$

b) $f(x) = (6\log_2 x)(\arcsin x)$

b) $f'(x) = \frac{6}{(\ln 2)x} \cdot \arcsin x + 6\log_2 x \cdot \frac{1}{\sqrt{1-x^2}}$

c) $f(x) = e^{\tanh x^3}$

c) $f'(x) = e^{\tanh x^3} \cdot \operatorname{sech}^2 x^3 \cdot 3x^2$

d) $f(x) = \frac{4^x}{\sinh x}$

d) $f'(x) = \frac{(\ln 4)4^x \cdot \sinh x - 4^x \cdot \cosh x}{\sinh^2 x}$

(6)

x	f	g	f'	g'
-2	1	0	6	-5
0	-2	1	3	2
1	0	-2	-1	4

a) $\frac{d}{dx}\left(f(1)+4g(0)\right)$

a) $f'(1)+4g'(0)$

$-1+4(2)=\boxed{7}$

b) $\frac{d}{dx}\left(\frac{g(1)}{f(-2)}\right)$

b) $\frac{g'(1)f(-2)-g(1)f'(-2)}{(f(-2))^2}$

$\frac{4\cdot 1 - -2\cdot 6}{1^2}=\boxed{16}$

c) $\frac{d}{dx}\left(g(f(0))\right)$

c) $g'(f(0))\cdot f'(0)$

$g'(-2)\cdot 3$

$-5\cdot 3$

-15

d) $\frac{d}{dx}\left((f(1))^3\right)$

d) $3(f(1))^2\cdot f'(1)$

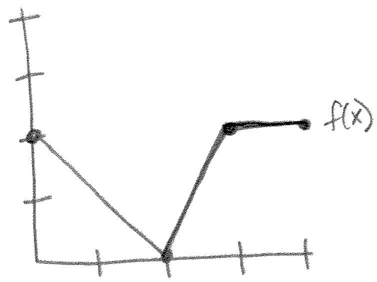
$3(0)^2\cdot -1$

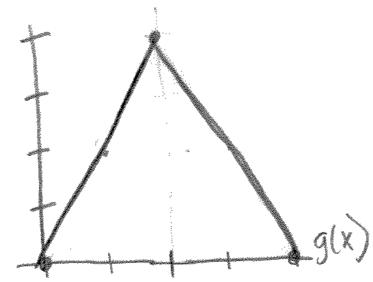
0

e) $\frac{d}{dx}\left(g^{-1}(-2)\right)$

e) $\frac{1}{g'(1)}=\boxed{\frac{1}{4}}$

⑦





a) If $h(x) = f(x) - g(x)$, find $h'(1)$.

b) If $p(x) = f(x) \cdot g(x)$, find $p'(1)$

c) If $m(x) = f(g(x))$, find $m'(1)$.

a) $f'(1) - g'(1)$

$-1 - 2 = -3$

b) $f'(1)g(1) + f(1)g'(1)$

$-1 \cdot 2 + 1 \cdot 2$

0

c) $f'(g(1)) \cdot g'(1)$

$f'(2) \cdot 2$

und $\cdot 2$ = undefined