A.P. Calculus — *Chapter 3 Review*

1. Find the derivative of the following:

(a) $y = \sin\left(e^x + 5x\right)$ (b) $y = \arctan\left(7^x\right)$ (c) $y = \arcsin(\tan x)$

(d) $y = \ln(x)\cos(x)$ (e) $y = \dfrac{\cosh(x)}{x^3 - 8}$ (f) $y = \sinh\left(e^{x^2+1}\right)$

(g) $y = x^2 \tanh(x)$ (h) $y = e^{\left(8x^3-7x\right)^4}$ (i) $y = \log_2\left(7x^2 - 8x + 2\right)$

2. Use the equation $x^3 - 2xy + y^2 = 5$ to answer the following.

(a) Find the expression for $\dfrac{dy}{dx}$.

(b) What is the equation of the tangent line at the point (2, 3)?

3. Given the bob at the end of a pendulum swings with a motion defined by $f(t) = 2 + 3\sin(t)$ where t is time in seconds and $f(t)$ is the height of the bob measured in centimeters.

(a) Find the height of the bob at 2 seconds.

(b) Find the velocity of the bob at 2 seconds.

(c) Find the acceleration of the bob at 2 seconds.

(d) Find the jerk of the bob at 2 seconds.

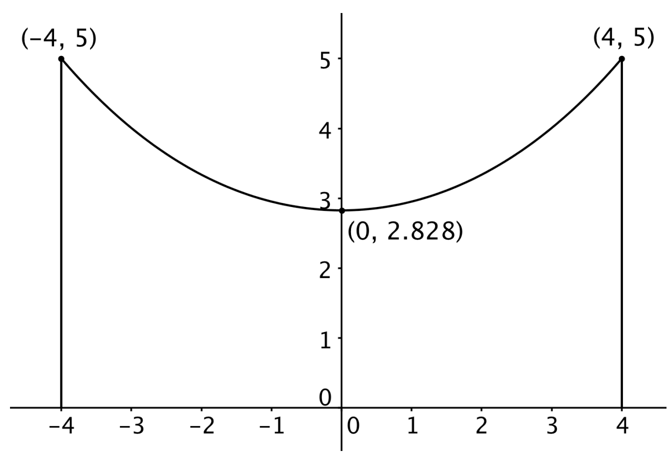
4. (a) Find the linearization function $L(x)$ for $f(x) = x^4$ at $x = 2$.

(b) Use your answer from part (a) to approximate $f(2.1)$.

(c) Find the error in the approximation.

5. Find $\dfrac{d}{dx} f^{-1}(9)$ when $f(x) = x^3 + \frac{2}{x}$.

6. Two 5-foot tall poles, that are 8 feet apart, have a chain suspended between them. The center of the chain is 2.828 feet above the ground. Find the equation of the catenary curve in the form $y = b + a\cosh\left(\frac{x}{a}\right)$ that represents the chain.



7. Use the chart below to find the following derivatives.

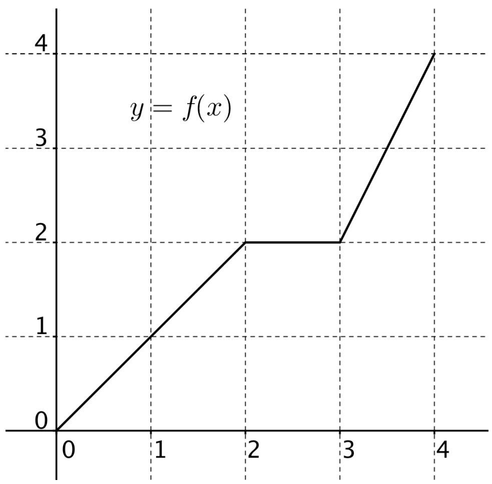
x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	-5	8
2	3	1	7	-4
3	1	2	-2	6

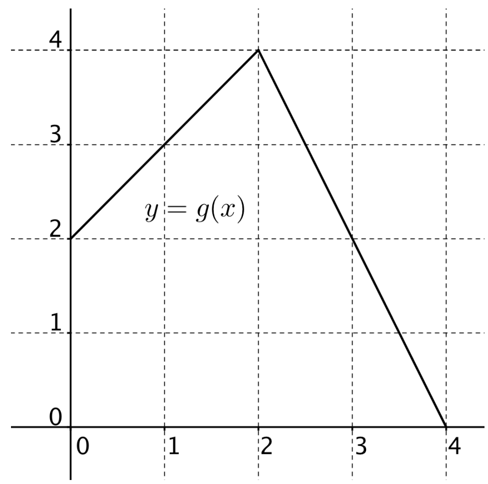
(a) $\frac{d}{dx}\left(f(3)+g(3)\right)$ (b) $\frac{d}{dx}\left(f(2)\cdot g(2)\right)$ (c) $\frac{d}{dx}\left(\frac{f(3)}{g(3)}\right)$

(d) $\frac{d}{dx}\left(g(f(2))\right)$ (e) $\frac{d}{dx}\left(f(g(1))\right)$ (f) $\frac{d}{dx}\left((f(1))^3+x\right)$

(g) $\frac{d}{dx}\left(\sqrt{g(2)}\right)$ (h) $\frac{d}{dx}\left(f^{-1}(3)\right)$ (i) $\frac{d}{dx}\left(g^{-1}(3)\right)$

8. Use the graphs below to answer the following.





(a) If $h(x) = f(x) + g(x)$, then find $h'(1)$ (b) If $k(x) = f(x)\ g(x)$, then find $k'(1)$

(c) If $p(x) = f(x) / g(x)$, then find $p'(1)$ (d) If $q(x) = x^2\ g(x)$, then find $q'(3)$

A.P. Calculus — *Chapter 3 Review Answers*

1a. $\frac{dy}{dx}=\cos\left(e^x+5x\right)\left(e^x+5\right)$

1b. $\frac{dy}{dx}=\frac{(\ln 7)\left(7^x\right)}{1+\left(7^x\right)^2}$

1c. $\frac{dy}{dx}=\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$

1d. $\frac{dy}{dx}=\left(\frac{1}{x}\right)\cos(x)+\ln(x)(-\sin(x))$

1e. $\frac{dy}{dx}=\frac{\sinh(x)\left(x^3-8\right)-\cosh(x)\left(3x^2\right)}{\left(x^3-8\right)^2}$

1f. $\frac{dy}{dx}=\cosh\left(e^{x^2+1}\right)\left(e^{x^2+1}\right)(2x)$

1g. $\frac{dy}{dx}=2x\tanh(x)+x^2\left(\frac{1}{\cosh^2 x}\right)$

1h. $\frac{dy}{dx}=e^{\left(8x^3-7x\right)^4}4\left(8x^3-7x\right)^3\left(24x^2-7\right)$

1i. $\frac{dy}{dx}=\frac{14x-8}{\ln 2\left(7x^2-8x+2\right)}$

2a. $3x^2-2y-2x\frac{dy}{dx}+2y\frac{dy}{dx}=0 \Rightarrow 3x^2-2y=(2x-2y)\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{3x^2-2y}{2x-2y}$

2b. $m=\frac{3(2)^2-2(3)}{2(2)-2(3)}=\frac{6}{-2}=-3$, equation of tangent line is $y-3=-3(x-2)$ or $y=-3x+9$

3a. Height $=f(2)=2+3\sin(2)=4.728\ cm$

3b. Velocity = $f'(2)=3\cos(2)=-1.248\ \frac{cm}{\text{sec}}$

3c. Acceleration = $f''(2)=-3\sin(2)=-2.728\ \frac{cm}{\text{sec}^2}$

3d. Jerk = $f'''(2)=-3\cos(2)=1.248\ \frac{cm}{\text{sec}^3}$

4a. $f(2)=(2)^4=16$, $f'(x)=4x^3$, therefore $m=4(2)^3=32$, then $L(x)=16+32(x-2)$.

4b. $L(2.1)=16+32(2.1-2)=16+32(0.1)=16+3.2=19.2$

4c. Actual is $f(2.1)=(2.1)^4=19.4481$, therefore error = 19.4491 – 19.2 = .2481

5. $f^{-1}(9)$ gives $9=x^3+\frac{2}{x}$, so $x=2$. $f'(x)=3x^2-\frac{2}{x^2}$ gives $f'(2)=3(2)^2-\frac{2}{2^2}=\frac{23}{2}$, then $\frac{d}{dx}f^{-1}(9)=\frac{2}{23}$.

6. Using point (0, 2.828) gives $2.828 = b + a\cosh\left(\frac{0}{a}\right)$, therefore $2.828 = b + a$ or $2.828 - a = b$. Then the point (4, 5) gives $5 = 2.828 - a + a\cosh\left(\frac{4}{a}\right)$ or $0 = -2.172 - a + a\cosh\left(\frac{4}{a}\right)$. Using the zero function on the calculator, $a = 4$. Then $b = 2.828 - 4 = -1.172$. Equation is $y = -1.172 + 4\cosh\left(\frac{x}{4}\right)$.

7a. $\frac{d}{dx}\left(f(3) + g(3)\right) = f'(3) + g(3) = -2 + 6 = 4$

7b. $\frac{d}{dx}\left(f(2)\cdot g(2)\right) = f'(2)g(2) + f(2)g'(2) = (7)(1) + (3)(-4) = -5$

7c. $\frac{d}{dx}\left(\frac{f(3)}{g(3)}\right) = \frac{f'(3)g(3) - f(3)g'(3)}{\left(g(3)\right)^2} = \frac{(-2)(2) - (1)(6)}{(2)^2} = \frac{-5}{2}$

7d. $\frac{d}{dx}\left(g\left(f(2)\right)\right) = g'\left(f(2)\right)f'(2) = g'(3)f'(2) = (6)(7) = 42$

7e. $\frac{d}{dx}\left(f\left(g(1)\right)\right) = f'\left(g(1)\right)g'(1) = f'(3)g'(1) = (-2)(8) = -16$

7f. $\frac{d}{dx}\left(\left(f(1)\right)^3 + x\right) = 3\left(f(1)\right)^2\left(f'(1)\right) + 1 = 3(2)^2(-5) + 1 = -60 + 1 = -59$

7g. $\frac{d}{dx}\left(\sqrt{g(2)}\right) = \frac{d}{dx}\left(g(2)\right)^{\frac{1}{2}} = \frac{1}{2}\left(g(2)\right)^{\frac{-1}{2}}\left(g'(2)\right) = \frac{1}{2}(1)^{\frac{-1}{2}}(-4) = -2$

7h. $\frac{d}{dx}\left(f^{-1}(3)\right) = \frac{1}{f'(2)} = \frac{1}{7}$

7i. $\frac{d}{dx}\left(g^{-1}(3)\right) = \frac{1}{g'(1)} = \frac{1}{8}$

8a. $h'(1) = f'(1) + g'(1) = 1 + 1 = 2$

8b. $k'(1) = f'(1)g(1) + f(1)g'(1) = (1)(3) + (1)(1) = 4$

8c. $p'(1) = \frac{f'(1)g(1) - f(1)g'(1)}{\left(g(1)\right)^2} = \frac{(1)(3) - (1)(1)}{(3)^2} = \frac{2}{9}$

8d. $q'(x) = 2xg(x) + x^2g'(x)$, then $q'(3) = 2(3)g\left((3)\right) + (3)^2g'\left((3)\right) = (6)(2) + (9)(-2) = -6$