Unit 6.3-6.9 Exponential and Logarithmic Functions

By the end of this unit, you should be able to:

- ☐ evaluate exponential functions
- ☐ graph exponential functions using transformations
- ☐ define and use the number e
- ☐ solve exponential equations
- ☐ convert between exponential expressions and logarithmic expressions
- ☐ evaluate logarithmic expressions
- ☐ graph logarithmic expressions
- ☐ solve logarithmic equations
- □ work with properties of logarithms
- ☐ evaluate logarithms using the change-of-base formula
- ☐ use the simple interest formula
- □ calculate effective rates of return
- ☐ use compound interest formula
- ☐ use continuously compound interest formula
- ☐ use the law of uninhibited growth
- □ use Newton's Law of Cooling
- □ use logistic models
- ☐ use a graphing calculator to fit exponential, logarithmic, and logistic functions to data

Assignments:

- 6.4 pg. 446 #9-35odd, 59, 75abc, 76abc, 79abc, 81abc, 87-109odd
- 6.5 pg. 457 #7-21odd, 31, 37, 41, 43, 45, 51, 55, 59, 61, 63, 65-71odd
- 6.6 pg. 463 #7, 9, 11, 15, 21, 23, 26, 29, 33, 39, 41, 43, 45, 49, 51, 53
- 6.7 pg. 472 #1, 5, 7, 11, 13, 15, 19, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 45, 49
- 6.8 pg. 484 #1, 5, 7, 9, 11, 13, 14, 17, 23, 26
- 6.9 pg. 491 #3, 5, 7, 9, 11

Review Problems

1. Convert the logarithmic expression into an equivalent expression using an exponent.

a)
$$\log_{7} 4 = y$$

b)
$$\log_5(x+4) = 3$$

2. Convert exponential expression into an equivalent expression using a logarithm.

a)
$$b^{x} = 5$$

b)
$$5^{x+4} = 8$$

3. Determine the exact value

a)
$$\log_4 256$$

b)
$$\log_2 \frac{1}{8}$$

4. Write the expression as a single logarithm. Express rational powers as roots.

a)
$$\log_2 10 - \frac{1}{3} \log_2 8 - 3 \log_2 v$$

a)
$$\log_2 10 - \frac{1}{3} \log_2 8 - 3 \log_2 v$$
 b) $2 \log_2 (b-1) - \frac{1}{2} \log_2 (4b^2 + 9) + \log_2 7b$

People have 10 fingers and 10 toes.

It makes sense to use base 10.

Natural-e.

5. Write the expression as the sum or difference of logarithms. Express powers or roots as factors.

a)
$$\ln \left(\frac{a^3bc^2}{\left(d-2\right)^4} \right)$$

$$b) \log_4 \frac{\sqrt{x-2}}{x+1}$$

6. Graph by using parent function and transformations. Find the domain, range, and asymptote(s).

a)
$$f(x) = 3\left(\frac{1}{2}\right)^{-x} + 5$$

b)
$$f(x) = \frac{1}{2}\log_3(x-2) + 1$$

7. Solve each equation for all real solutions. Express in exact form and approximate solutions (rounded to 3 decimal places).

a)
$$3^{-2x} = 27^{x-2}$$

b)
$$\log_4 x + \log_4 (x+3) = 1$$

c)
$$6 = 4^{4x}$$

d)
$$\ln(10x) = \ln(x-2) + 1$$
 e) $4^{2x} - 4 \cdot 4^x - 5 = 0$

e)
$$4^{2x} - 4 \cdot 4^x - 5 = 0$$

c)
$$6 = 4^{4x}$$

f) $5^{x+2} = 7^{x-2}$

8. Evaluate each logarithm. Round to 4 decimal places.

a)
$$\log_4 12$$

b)
$$\log_{1.3} \sqrt{14}$$

- 9. Find the value of \$200 invested at 10% interest compounded quarterly for 8 years.
- 10. How long will it take for \$200 to grow to \$1,500 with a continuously compounded rate of 6.2%?
- 11. What is the initial investment required if an account grows to \$3,000 at 5.3% interest compounded monthly in 10 years?
- 12. How long does it take for \$100 to triple in value if the interest is compounded weekly at 6%?
- 13. The number of bacteria on a countertop after t hours is given by the equation $N = 950e^{0.04t}$
 - a) How many bacteria are present after 6 hours?
 - b) How long will it take for the count to be 5,000 bacteria?
- 14. There were 900 Polar Bears in the wild in 1972. In 1997, there were 2045 bears.
 - a) Write an equation for the number of bears t years after 1972 assuming uninhibited growth.
 - b) Use the equation found in part a to estimate the number of Polar Bears in 2015.
- 15. A skillet is removed from an oven whose temperature is 450 F and placed in a room whose temperature is 70 F. After 5 minutes, the temperature of the skillet is 400 F. How long will it be until its temperature is 150 F?
- 16. At North Shore High School, Regina, Gretchen, Karen, and Cady start a rumor which spreads logistically so that $S(t) = \frac{1200}{1+39e^{-0.9t}}$ models the number of students at the school who have heard

the rumor by the end of t days, where t = 0 is the day the rumor begins to spread.

- a) What is the population of the school?
- b) How many students have heard the rumor after 4 days?
- c) How long does it take for 1000 student to hear the rumor?
- 17. Which model (exponential, logarithmic, or logistic) best models the population of Dallas, TX? Find the model. Use it to predict the 2020 population.

Year	Population
1950	434,462
1960	679,684
1970	844,401
1980	904,599
1990	1,006,877
2000	1,188,589

Source: U.S. Census Bureau.