

# 6.3-6.9 Review Key

① a)  $7^y = 4$

b)  $5^3 = x+4$

② a)  $\log_b 5 = x$

b)  $\log_5 8 = x+4$

③ a)  $\log_4 256 = \boxed{4}$

$$4^? = 256$$

b)  $\log_2 \frac{1}{8} = \boxed{-3}$

$$2^? = \frac{1}{8}$$

④ a)  $\log_2 10 - \frac{1}{3} \log_2 8 - 3 \log_2 v$

$$\log_2 10 - \log_2 \sqrt[3]{8} - \log_2 v^3$$

$$\log_2 10 - \log_2 2 - \log_2 v^3$$

$$\log_2 10 - 1 - \log_2 v^3$$

$$\log_2 10 - \log_2 v^3 - 1$$

$$\boxed{\log_2 \frac{10}{v^3} - 1}$$

b)  $2 \log_2(b-1) - \frac{1}{2} \log_2(4b^2+9) + \log_2 7b$

$$\log_2(b-1)^2 - \log_2 \sqrt{4b^2+9} + \log_2 7b$$

$$\log_2 \frac{(b-1)^2}{\sqrt{4b^2+9}} + \log_2 7b$$

$$\boxed{\log_2 \frac{(b-1)^2 \cdot 7b}{\sqrt{4b^2+9}}}$$

$$(5) \text{ a) } \ln \left( \frac{a^3 b c^2}{(d-2)^4} \right)$$

$$\ln a^3 + \ln b + \ln c^2 - \ln(d-2)^4$$

$$\boxed{3\ln a + \ln b + 2\ln c - 4\ln(d-2)}$$

$$\text{b) } \log_4 \frac{\sqrt{x-2}}{x+1}$$

$$\log_4 \sqrt{x-2} - \log_4 (x+1)$$

$$\boxed{\frac{1}{2} \log_4 (x-2) - \log_4 (x+1)}$$

$$(6) \text{ a) } f(x) = 3 \left(\frac{1}{2}\right)^{-x} + 5$$

parent:  $y = \left(\frac{1}{2}\right)^x$

transformations:

vertical stretch by 3

reflection over y-axis

vertical shift up 5

x	y
2	-2
-1	-1
0	0
-2	-2
1	2
3	6
8	17
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{3}{2}$
$\frac{6.5}{2}$	$\frac{6.5}{2}$
$\frac{5.75}{2}$	$\frac{5.75}{2}$
$\frac{3}{4}$	$\frac{3}{4}$

$$\text{b) } f(x) = \frac{1}{2} \log_3 (x-2) + 1$$

parent:  $y = \log_3 x$

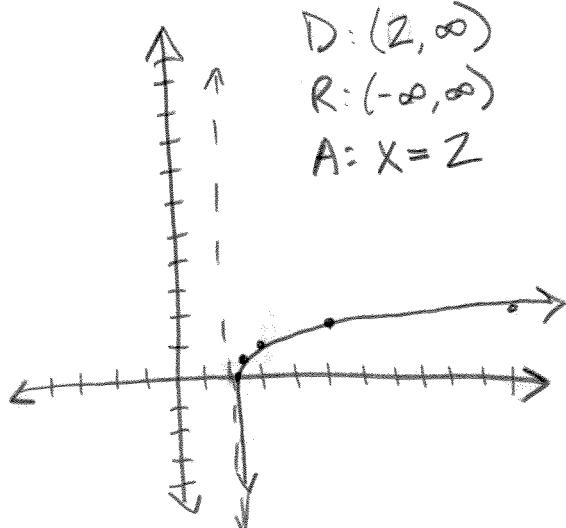
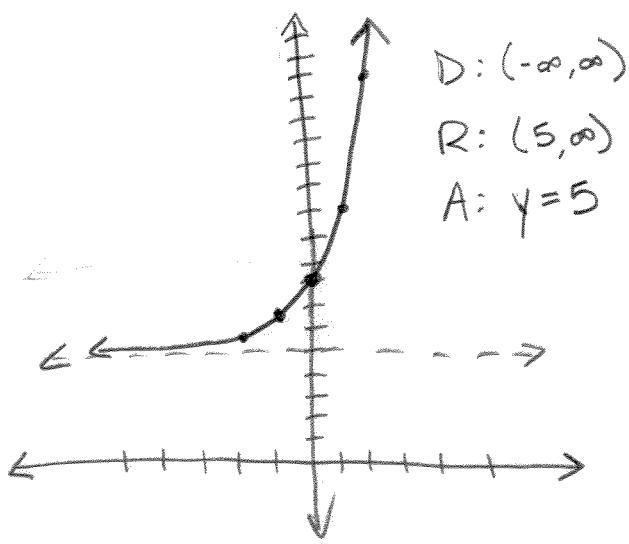
transformations:

vertical shrink by  $\frac{1}{2}$

horizontal shift right 2

vertical shift up 1

x	y
$2\frac{1}{3}$	$\frac{1}{3}$
$2\frac{2}{3}$	$\frac{1}{2}$
3	1
5	$\frac{3}{2}$
11	2
$2\frac{1}{2}$	$\frac{1}{2}$
$2\frac{1}{3}$	$\frac{1}{3}$
3	0
5	$\frac{1}{2}$
11	1



$$\textcircled{1} \quad a) \quad 3^{-2x} = 27^{x-2}$$

$$3^{-2x} = (3^3)^{x-2}$$

$$3^{-2x} = 3^{3x-6}$$

$$-2x = 3x - 6$$

$$-3x \quad -3x$$

$$\frac{-5x}{-5} = \frac{-6}{-5}$$

$$\boxed{x = \frac{6}{5}}$$

$$b) \log_4 x + \log_4 (x+3) = 1$$

$$\log_4(x(x+3)) = 1$$

$$\log_4(x^2 + 3x) = 1$$

$$4^1 = x^2 + 3x$$

$$0 = x^2 + 3x - 4$$

$$0 = (x+4)(x-1)$$

$$\cancel{x \neq -4} \quad \boxed{x=1}$$

$$c) \quad 6 = 4^{4x}$$

$$\log_4 6 = 4x$$

$$\left(\frac{1}{4}\right) \frac{\log 6}{\log 4} = 4x \left(\frac{1}{4}\right)$$

$$\boxed{\frac{\log 6}{4 \log 4} = x} \quad \text{exact}$$

$$\boxed{x \approx 0.323} \quad \text{approx}$$

$$d) \quad \ln(10x) = \ln(x-2) + 1$$

$$\ln(10x) - \ln(x-2) = 1$$

$$\ln\left(\frac{10x}{x-2}\right) = 1$$

$$(x-2)e^1 = \frac{10x}{x-2}(x-2)$$

$$\begin{aligned} xe - 2e &= 10x \\ -xe &= -xe \\ -2e &= 10x - xe \\ -2e &= x(10 - e) \end{aligned}$$

$$\boxed{x = \frac{-2e}{10-e}} \quad \boxed{x \approx -0.747}$$

exact    approx

$$e) \quad 4^{2x} - 4 \cdot 4^x - 5 = 0 \quad \text{let } u = 4^x$$

$$(4^x)^2 - 4 \cdot 4^x - 5 = 0$$

$$u^2 - 4u - 5 = 0$$

$$(u+1)(u-5) = 0$$

$$u = -1 \quad u = 5$$

$$4^x \neq -1 \quad 4^x = 5$$

$$\log_4 5 = x$$

$$\boxed{\frac{\log 5}{\log 4} = x}$$

$\boxed{x \approx 1.161}$

$$f) \quad 5^{x+2} = 7^{x-2}$$

(13)

$$\log 5^{x+2} = \log 7^{x-2}$$

$$(x+2)\log 5 = (x-2)\log 7$$

$$x\log 5 + 2\log 5 = x\log 7 - 2\log 7$$

$$x\log 5 - x\log 7 = -2\log 5 - 2\log 7$$

$$x(\log 5 - \log 7) = \log 5^2 - \log 7^2$$

$$x(\log \frac{5}{7}) = \log \frac{5^2}{7^2}$$

$$\frac{x(\log \frac{5}{7})}{\log \frac{5}{7}} = \frac{\log \frac{1}{1225}}{\log \frac{5}{7}}$$

$$x = \frac{\log \frac{1}{1225}}{\log \frac{5}{7}}$$

$$x \approx 16.097$$

⑧ a)  $\log_4 12 = \frac{\log 12}{\log 4} \approx 1.7925$

b)  $\log_{1.3} \sqrt{14} = \frac{\log \sqrt{14}}{\log 1.3} \approx 5.6294$

⑨  $A = 200 \left(1 + \frac{1}{4}\right)^{(4 \cdot 8)}$

$$= 440.75$$

$$\textcircled{10} \quad \frac{1500}{200} = \frac{200}{200} e^{.062t}$$

$$\frac{15}{2} = e^{.062t}$$

$$\ln \frac{15}{2} = \frac{.062t}{.062}$$

$$t \approx 32.498 \text{ yrs.}$$

$$\textcircled{11} \quad \frac{3,000}{(1 + \frac{.053}{12})^{12 \cdot 10}} = P \left(1 + \frac{.053}{12}\right)^{12 \cdot 10}$$

$$P = \$1767.88$$

$$\textcircled{12} \quad 3 = \left(1 + \frac{.06}{52}\right)^{52t}$$

$$\log_{\left(1 + \frac{.06}{52}\right)} 3 = 52t$$

$$\frac{\log 3}{\log \left(1 + \frac{.06}{52}\right)} = \frac{52t}{52}$$

$$t \approx 18.32 \text{ yrs.}$$

$$\textcircled{13} \quad N = 950 e^{.04t}$$

$$\text{a) } N(6) = 950 e^{.04(6)} \\ = \$1207.69$$

$$\text{b) } 5,000 = 950 e^{.04t}$$

$$\frac{500}{95} = e^{.04t}$$

$$\frac{\ln \frac{500}{95}}{.04} = \frac{.04t}{.04}$$

$$t \approx 41.52 \text{ hrs}$$

$$\textcircled{14} \quad \text{a) } N(t) = N_0 e^{kt} \quad \text{Let } 1972 \Rightarrow t=0$$

$$\frac{2045}{900} = \frac{900 e^{k \cdot 25}}{900}$$

$$\frac{2045}{900} = e^{25k}$$

$$\ln \frac{2045}{900} = \frac{25k}{25}$$

$$k \approx .0328$$

$$N(t) = 900 e^{.0328t}$$

$$\text{b) } 2015 \Rightarrow 43 \text{ yrs.}$$

$$N(43) = 900 e^{.0328(43)}$$

$$= 3687 \text{ polar bears}$$

$$\textcircled{15} \quad u(t) = T + (u_0 - T)e^{-kt}$$

$$400 = 70 + (450 - 70)e^{-5k}$$

$$330 = 380 e^{-5k}$$

$$\frac{33}{38} = e^{-5k}$$

$$\ln \frac{33}{38} = \frac{5k}{5} \quad k \approx -.0282$$

$$150 = 70 + 380 e^{-0.0282t}$$

$$\frac{80}{380} = e^{-0.0282t}$$

$$\frac{\ln \frac{80}{380}}{-0.0282} = \frac{-0.0282t}{-0.0282}$$

$$t \approx 55.25 \text{ min}$$

⑯ a) 1200

b)  $S(4) = \frac{1200}{1 + 39e^{-0.9(4)}} \approx 580 \text{ people}$

c)  $1000 = \frac{1200}{1 + 39e^{-0.9t}}$

$$(1 + 39e^{-0.9t})(1000) = \frac{1200}{1000}$$

$$1 + 39e^{-0.9t} = 1.2$$

$$\frac{39e^{-0.9t}}{39} = \frac{11}{39}$$

$$e^{-0.9t} = \frac{11}{39}$$

$$\frac{\ln \frac{11}{39}}{-0.9} = \frac{-0.9t}{-0.9}$$

$$t \approx 1.41 \text{ days}$$

⑰ Let  $t=0 \Rightarrow 1950$

exponential

$$y = 513338.40 (1.0181)^x$$

$$y = 513338.40 (1.0181)^{70} \quad 2020 \Rightarrow 70 \text{ yrs.}$$

$$x \boxed{1,801,915}$$