

13.1-13.5 Review Key

1a) $\{b_n\} = \{(-1)^{n+1}(2n+3)\}$

$a_1 = (-1)^{1+1}(2(1)+3) = 5$

$a_2 = (-1)^{2+1}(2(2)+3) = -7$

$a_3 = (-1)^{3+1}(2(3)+3) = 9$

$a_4 = (-1)^{4+1}(2(4)+3) = -11$

$a_5 = (-1)^{5+1}(2(5)+3) = -13$

1b) $a_1 = -3$ $a_n = 2 - a_{n-1}$

$a_2 = 2 - a_1 = 2 - (-3) = 5$

$a_3 = 2 - a_2 = 2 - 5 = -3$

$a_4 = 2 - a_3 = 2 - (-3) = 5$

$a_5 = 2 - a_4 = 2 - 5 = -3$

2) $\sum_{k=1}^3 (3-k^2) = \boxed{2 + -1 + -6}$

3) $2 + \frac{2^2}{3} + \frac{2^3}{3^2} + \dots + \frac{2^{n+1}}{3^n} =$
 $\sum_{k=0}^n \frac{2^{k+1}}{3^k}$

4a) $\{b_n\} = \{4n+3\}$

arithmetic

Common difference of 4

4b) $\{d_n\} = \{2n^2-1\}$

neither

4c) $\{u_n\} = 3^{2n}$

geometric

common ratio of 9

5a) $\sum_{k=1}^{40} (-2k+8)$

arithmetic sum... so $S_n = \frac{n}{2}(a_1 + a_n)$

$S_{40} = \frac{40}{2}(6 + -12) = \boxed{-1320}$

5b) $-5, -1, 3, 7, \dots, 363$

arithmetic sum... so $S_n = \frac{n}{2}(a_1 + a_n)$

of terms?

$a_n = a_1 + (n-1)d$

$363 = -5 + (n-1)4$

$+5 \quad +5$

$\frac{368}{4} = \frac{(n-1)4}{4}$

$92 = n-1$

$+1 \quad +1$

$n = 93$

$S_{93} = \frac{93}{2}(-5 + 363) = \boxed{16647}$

6. $1, -1, -3, -5, \dots$

arithmetic

$$a_n = 1 + (n-1)(-2)$$

$$a_n = -2n + 3$$

$$a_{98} = -2(98) + 3 = \boxed{-193}$$

7. $1, \frac{1}{10}, \frac{1}{100}$

geometric

$$a_n = 1 \cdot \left(\frac{1}{10}\right)^{n-1}$$

$$a_9 = 1 \cdot \left(\frac{1}{10}\right)^{9-1} = \boxed{.00000008}$$

8a) $\frac{1}{3}, \frac{4}{3}, \frac{9}{3}, \frac{16}{3}, \frac{25}{3}, \dots$

$$a_n = \frac{n^2}{3}$$

8b) $-\frac{1}{4}, \frac{1}{8}, -\frac{1}{12}, \frac{1}{16}, -\frac{1}{20}, \dots$

$$a_n = \frac{(-1)^n}{4n}$$

9) $a_8 = -20$

$$a_{17} = -47$$

$$-20 = a_1 + (8-1)d$$

$$-47 = a_1 + (17-1)d$$

$$-(-20 = a_1 + 7d)$$

$$-47 = a_1 + 16d$$

$$\frac{-27 = 9d}{9 \quad 9}$$

$$d = -3$$

$$-20 = a_1 + 7(-3)$$

$$-20 = a_1 - 21$$

$$+21 \quad +21$$

$$1 = a_1$$

$$a_n = 1 + (n-1)(-3)$$

$$a_n = 1 - 3n + 3$$

$$a_n = \boxed{-3n + 4}$$

10a) $6 - 4 + \frac{8}{3} - \frac{16}{9} + \dots$

$$r = -\frac{2}{3} \Rightarrow \text{converges}$$

$$S_n = \frac{a_1}{1-r}$$

$$S_n = \frac{6}{1 - (-\frac{2}{3})} = \frac{6}{\frac{5}{3}} = 6 \cdot \frac{3}{5} = \boxed{\frac{18}{5}}$$

10b) $\sum_{k=1}^{\infty} 4\left(\frac{1}{2}\right)^{k-1}$

$$\text{converges} \Rightarrow r = \frac{1}{2}$$

$$S_n = \frac{a_1}{1-r} = \frac{4}{1 - \frac{1}{2}} = \frac{4}{\frac{1}{2}}$$

$$= 4 \cdot 2 = \boxed{8}$$

$$11) \quad 3 + 6 + 9 + \dots + 3n = \frac{3n}{2}(n+1)$$

Base case $n=1$

$$3 = \frac{3(1)}{2}(1+1)$$

$$3 = \frac{3}{2}(2)$$

$$3 = 3 \quad \checkmark$$

Assume for $k=1$

$$3 + 6 + 9 + \dots + 3k = \frac{3k}{2}(k+1)$$

Need to prove for $k+1$

$$\underline{3 + 6 + 9 + \dots + 3k} + 3(k+1) = \frac{3(k+1)}{2}(k+1+1)$$

$$\frac{3k}{2}(k+1) + 3(k+1) = \frac{3(k+1)}{2}(k+1+1)$$

(factor out $k+1$)

$$(k+1) \left(\frac{3k}{2} + 3 \right) = \frac{3(k+1)}{2}(k+1+1)$$

$$(k+1) \left(\frac{3k+6}{2} \right) = \frac{3(k+1)}{2}(k+1+1)$$

$$(k+1) \cdot \frac{3(k+2)}{2} = \frac{3(k+1)}{2}(k+1+1)$$

$$(k+1) \left(\frac{3}{2} \right) (k+2) = \frac{3(k+1)}{2}(k+1+1)$$

$$\frac{3(k+1)}{2}(k+1+1) = \frac{3(k+1)}{2}(k+1+1)$$

\checkmark

$$12) \quad (2x-3)^8$$

$$\binom{8}{5} (2x)^3 (-3)^5$$

$$56 \cdot 8x^3 \cdot -243$$

$$-108864x^3$$

$$\Rightarrow \boxed{-108864}$$

$$13) \quad A = R \left(\frac{(1 + \frac{r}{n})^{nt} - 1}{\frac{r}{n}} \right)$$

$$A = 200 \left(\frac{(1 + \frac{1}{12})^{12 \cdot 20} - 1}{\frac{1}{12}} \right) \approx \boxed{\$151,873.77}$$

$$14) \quad a_n = 20000 (1.04)^{n-1}$$

$$a_5 = 20,000 (1.04)^4 \approx \boxed{\$23,397.17}$$

$$15) \quad .123123123123\dots$$

$$.123 + .000123 + .000000123 + \dots$$

$$\sum_{k=1}^{\infty} .123 \left(\frac{1}{1000} \right)^{k-1} = \frac{a_1}{1-r} = \frac{.123}{1 - \frac{1}{1000}}$$

$$= \frac{.123}{\frac{999}{1000}} = .123 \cdot \frac{1000}{999} = \boxed{\frac{123}{999}}$$