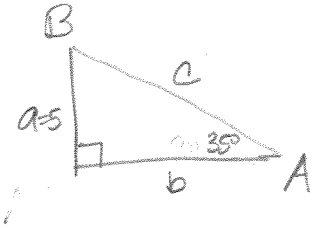
9.1-9.4 Review Key

① $a=5 \quad A=35^\circ \quad C=90^\circ$



$\sin 35^\circ = \frac{5}{c}$

$c = \frac{5}{\sin 35^\circ}$

$\boxed{c \approx 8.72}$

$B = 180-90-35$

$\boxed{B = 55^\circ}$

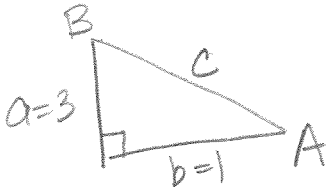
$5^2 + b^2 = 8.72^2$

$25 + b^2 = 76.0384$

$b^2 = 51.0384$

$\boxed{b \approx 7.14}$

② $a=3 \quad b=1 \quad C=90^\circ$



$\tan^{-1}(\tan A) = \tan^{-1}\left(\frac{3}{1}\right)$

$\boxed{A \approx 71.57^\circ}$

$B = 180^\circ - 90^\circ - 71.57^\circ$

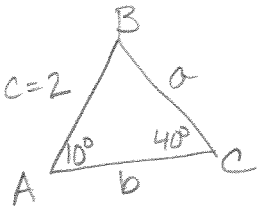
$\boxed{B \approx 18.43^\circ}$

$3^2 + 1^2 = c^2$

$10 = c^2$

$\boxed{c = \sqrt{10} \approx 3.16}$

③ $A=10^\circ \quad C=40^\circ \quad c=2$



AAS

$B = 180-10-40$

$\boxed{B = 130^\circ}$

$\frac{\sin 40^\circ}{2} = \frac{\sin 10^\circ}{a}$

$\frac{a \sin 40^\circ}{\sin 40^\circ} = \frac{2 \sin 10^\circ}{\sin 40^\circ}$

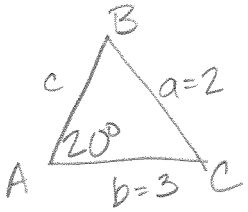
$\boxed{a \approx .54}$

$\frac{\sin 40^\circ}{2} = \frac{\sin 130^\circ}{b}$

$\frac{b \sin 40^\circ}{\sin 40^\circ} = \frac{2 \sin 130^\circ}{\sin 40^\circ}$

$\boxed{b \approx 2.38}$

④ $a=2 \quad b=3 \quad A=20^\circ$



SSA

$\frac{\sin 20^\circ}{2} = \frac{\sin B}{3}$

$\frac{2 \sin B}{2} = \frac{3 \sin 20^\circ}{2}$

$\sin^{-1}(\sin B) = \sin^{-1}\left(\frac{3 \sin 20^\circ}{2}\right)$

$\boxed{B = 30.87}$

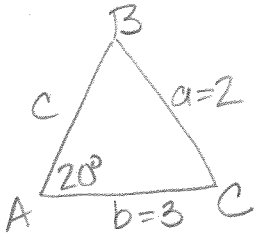
$\frac{\sin 20^\circ}{2} = \frac{\sin 129.13^\circ}{c}$

$\frac{c \sin 20^\circ}{\sin 20^\circ} = \frac{2 \sin 129.13}{\sin 20^\circ}$

$\boxed{c \approx 4.54}$

$C = 180-20-30.87$

$\boxed{C = 129.13^\circ}$



$B = 180-30.87$

$\boxed{B \approx 149.13^\circ}$

$C = 180-20-149.13$

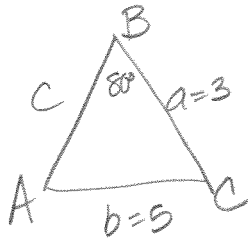
$\boxed{C \approx 10.87^\circ}$

$\frac{\sin 10.87^\circ}{c} = \frac{\sin 20^\circ}{2}$

$\frac{c \sin 20^\circ}{\sin 20^\circ} = \frac{2 \sin 10.87^\circ}{\sin 20^\circ}$

$\boxed{c \approx 1.10}$

⑤ $a=3 \quad b=5 \quad B=80^\circ$



SSA

$$\frac{\sin 80^\circ}{5} = \frac{\sin A}{3}$$

$$\frac{5\sin A}{5} = \frac{3\sin 80^\circ}{5}$$

$$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{3\sin 80^\circ}{5}\right)$$

$$\boxed{A = 36.22^\circ}$$

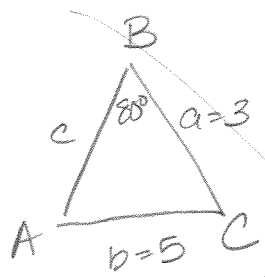
$$C = 180^\circ - 36.22 - 80^\circ$$

$$\boxed{C \approx 63.78^\circ}$$

$$\frac{\sin 80^\circ}{5} = \frac{\sin 63.78^\circ}{c}$$

$$\frac{c\sin 80^\circ}{\sin 80^\circ} = \frac{5\sin 63.78^\circ}{\sin 80^\circ}$$

$$\boxed{c \approx 4.55}$$

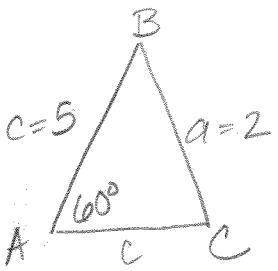


$$A = 180^\circ - 36.22^\circ$$

$$A \approx 143.78^\circ$$

too big

⑥ $a=2 \quad c=5 \quad A=60^\circ$



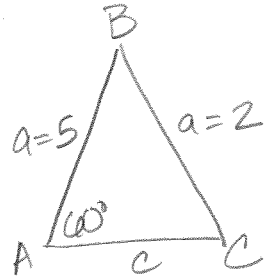
SSA

$$\frac{\sin 60^\circ}{2} = \frac{\sin C}{5}$$

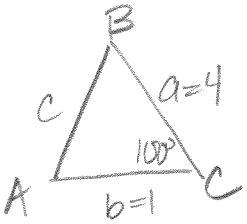
$$\frac{2\sin C}{2} = \frac{5\sin 60^\circ}{2}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{5}{2}\sin 60^\circ\right)$$

$$\boxed{\text{no triangle}}$$



⑦ $a=4 \quad b=1 \quad C=100^\circ$



SAS

$$c^2 = 4^2 + 1^2 - 2(1)(4)\cos 100^\circ$$

$$\boxed{c \approx 4.29}$$

$$4^2 = 1^2 + 4.29^2 - 2(1)(4.29)\cos A$$

$$16 = 19.40 - 8.58\cos A$$

$$-3.4 = -8.58\cos A$$

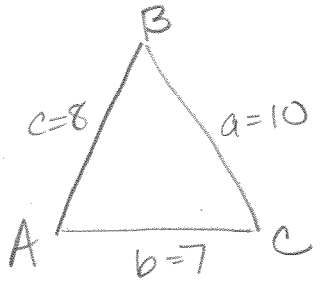
$$\cos^{-1}\left(\frac{3.4}{8.58}\right) = \cos^{-1}(\cos A)$$

$$\boxed{A \approx 66.65^\circ}$$

$$B = 180^\circ - 100^\circ - 66.65^\circ$$

$$\boxed{B = 13.35^\circ}$$

(8) $a=10$ $b=7$ $c=8$



SSS

$10^2 = 8^2 + 7^2 - 2(8)(7)\cos A$

$100 = 113 - 112\cos A$

$-13 = -112\cos A$

$\cos^{-1}\left(\frac{13}{112}\right) = \cos^{-1}(\cos A)$

$\boxed{83.33° = A}$

$8^2 = 10^2 + 7^2 - 2(10)(7)\cos C$

$64 = 149 - 140\cos C$

$-85 = -140\cos C$

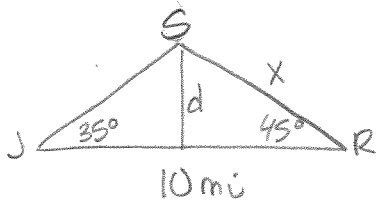
$\cos^{-1}\left(\frac{85}{140}\right) = \cos^{-1}(\cos C)$

$\boxed{C \approx 52.62°}$

$B = 180° - 52.62° - 83.33°$

$\boxed{B = 44.05°}$

(9)



$S = 180° - 45° - 35°$

$S = 100°$

$\frac{\sin 35°}{x} = \frac{\sin 100°}{10}$

$\frac{x\sin 100°}{\sin 100} = \frac{10\sin 35°}{\sin 100}$

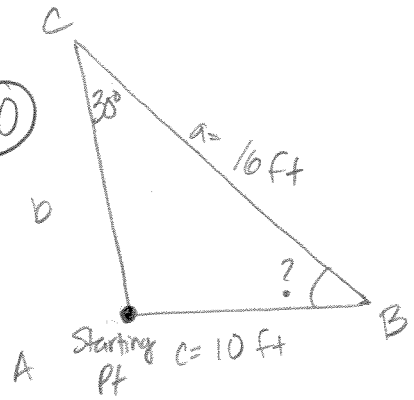
$x \approx 5.82$ mi

$\sin 45° = \frac{d}{5.82}$

$d = 5.82\sin 45°$

$\boxed{d = 4.12\text{mi}}$

(10)



$\frac{\sin 30°}{10} = \frac{\sin A}{16}$

$\frac{10\sin A}{10} = \frac{16\sin 30°}{10}$

$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{16\sin 30°}{10}\right)$

$A \approx 53.13°$

$B = 180° - 53.13° - 30°$

$B = 96.87°$

-or-

$A = 180 - 53.13°$

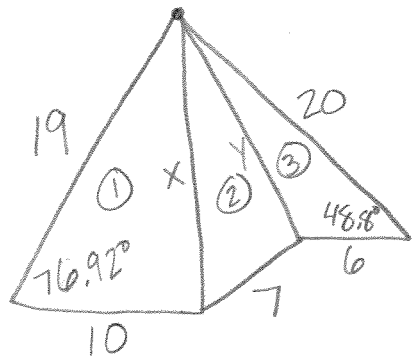
$A = 126.87°$

$B = 180° - 30° - 126.87°$

$B = 23.13°$

There is not enough information to determine who is correct. Both answers are plausible. Since we are given 2 sides and an angle not included (SSA), there could be 2 possible answers.

(11)



Triangle 1 Area

$A_1 = \frac{1}{2}(19)(10)\sin 76.92°$

$A_1 = 92.54 \text{ units}^2$

Triangle 3 Area

$A_3 = \frac{1}{2}(6)(20)\sin 48.8°$

$A_3 = 45.14 \text{ units}^2$

$x^2 = 19^2 + 10^2 - 2(10)(19)\cos 76.92°$

$x \approx 19.36$

$y^2 = 20^2 + 6^2 - 2(20)(6)\cos 48.8°$

$y \approx 16.67$

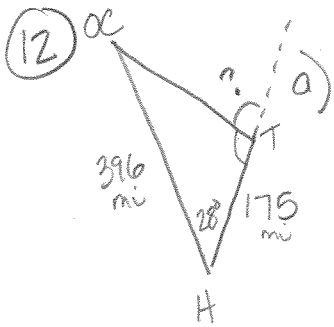
Triangle 2 Area

$s = \frac{1}{2}(19.36 + 16.67 + 7) \approx 21.515$

$A_2 = \sqrt{21.515(21.515 - 19.36)(21.515 - 16.67)(21.515 - 7)} \approx 57.10 \text{ units}^2$

Total Area: $A_1 + A_2 + A_3 = \boxed{194.78 \text{ units}^2}$

(12)



a) $h^2 = 396^2 + 175^2 - 2(396)(175)\cos 28°$

$h \approx 255.08 \text{ mi}$

$396^2 = 255.08^2 + 175^2 - 2(255.08)(175)\cos T$

$156816 = 95690.81 - 89278\cos T$

$61125.19 = -89278\cos T$

$\cos^{-1}\left(\frac{61125.19}{-89278}\right) = \cos^{-1}(\cos T)$

$T \approx 93.93°$

$180 - 93.93$

$\boxed{86.07°}$

b) direct route = 396 mi

route with turn =

$175 + 255.08 = 430.08$

$430.08 - 396 \approx \boxed{34 \text{ mi further}}$

c) direct route time:

$d = rt \quad 396 = 175t$

$t = 2.26 \text{ hrs}$

route with turn:

before turn: 1 hr

after turn needs to be 1.26 hr

$d = rt$

$255.08 = r(1.26)$

$r = \boxed{202.44 \text{ mph}}$