8.3 - 8.5 Review Key

① $\cos 75^\circ$

$$\cos(30^\circ + 45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

-or-

$$\cos\left(\frac{150^\circ}{2}\right) = \pm\sqrt{\frac{1 + \cos 150^\circ}{2}}$$

$$= +\sqrt{\frac{1 + -\frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2 - \sqrt{3}}}{2}}$$

② $\tan\frac{\pi}{12}$

$$\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}$$

$$= \frac{(\sqrt{3}-1)}{(1+\sqrt{3})}\cdot\frac{(1-\sqrt{3})}{(1-\sqrt{3})}$$

$$= \frac{\sqrt{3}-3-1+\sqrt{3}}{1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-3}$$

$$= \frac{-4+2\sqrt{3}}{-2}$$

$$= \boxed{2-\sqrt{3}}$$

-or-

$$\tan\left(\frac{\frac{\pi}{6}}{2}\right) = \frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\cdot\frac{2}{2}$$

$$= \boxed{2-\sqrt{3}}$$

③ $\cos\frac{5\pi}{12}\cos\frac{7\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{7\pi}{12}$

$\cos\left(\frac{5\pi}{12}+\frac{7\pi}{12}\right)$

$\cos(\pi)$

$\boxed{-1}$

(4) $\dfrac{\tan 40^\circ - \tan 10^\circ}{1 + \tan 40^\circ \tan 10^\circ}$

$\tan(40^\circ - 10^\circ)$

$\tan 30^\circ$

$\boxed{\dfrac{\sqrt{3}}{3}}$

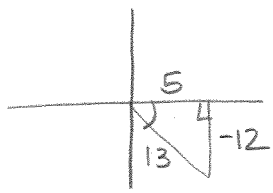
(5) $\cos^2 15^\circ - \sin^2 15^\circ$

$\cos(2 \cdot 15^\circ)$

$\cos 30^\circ$

$\boxed{\dfrac{\sqrt{3}}{2}}$

(6) $\sin\theta = \dfrac{-12}{13} \qquad \dfrac{3\pi}{2} < \theta < 2\pi$



$\cos\theta = \dfrac{5}{13}$

$\tan\theta = \dfrac{-12}{5}$

a) $\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2 \cdot \dfrac{-12}{13} \cdot \dfrac{5}{13}$

$= \boxed{\dfrac{-120}{169}}$

b) $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\dfrac{\frac{3\pi}{2}}{2} < \dfrac{\theta}{2} < \dfrac{2\pi}{2}$

$\dfrac{3\pi}{4} < \theta < \pi \Rightarrow QII$

$\Rightarrow \cos$ is $-$

$= -\sqrt{\dfrac{1+\frac{5}{13}}{2} \cdot \dfrac{13}{13}}$

$= -\sqrt{\dfrac{13+5}{26}}$

$= \dfrac{-\sqrt{18}}{\sqrt{26}} \cdot \dfrac{\sqrt{26}}{\sqrt{26}} = \boxed{\dfrac{-3\sqrt{52}}{26}}$

c) $\tan\left(\theta - \dfrac{\pi}{4}\right) = \dfrac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta\tan\frac{\pi}{4}} = \dfrac{\frac{-12}{5} - 1}{1 + \frac{-12}{5} \cdot 1} = \dfrac{\frac{-12}{5} - 1}{1 - \frac{12}{5}} \cdot \dfrac{5}{5}$

$= \dfrac{-12-5}{5-12} = \boxed{\dfrac{-17}{7}}$

(11) $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$\frac{\sin\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta} = \sec\theta\csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\sin\theta\cos\theta} = \sec\theta\csc\theta$$

$$\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta} = \sec\theta\csc\theta$$

$$\sec\theta\cdot\csc\theta = \sec\theta\csc\theta \quad \square$$

(12) $\cos(\pi-\theta) = -\cos\theta$

$$\cos\pi\cos\theta + \sin\pi\sin\theta = -\cos$$

$$-1\cos\theta + 0\sin\theta = -\cos\theta$$

$$-\cos\theta = -\cos\theta \checkmark$$

(13) the easy way

$$\frac{1+\sec\theta}{1+\cos\theta} = \sec\theta\cdot\frac{1+\cos\theta}{1+\cos\theta}$$

$$\frac{1+\sec\theta}{1+\cos\theta} = \frac{\sec\theta + \sec\theta\cos\theta}{1+\cos\theta}$$

$$\frac{1+\sec\theta}{1+\cos\theta} = \frac{\sec\theta+1}{1+\cos\theta}$$

$$\frac{1+\sec\theta}{1+\cos\theta} = \frac{1+\sec\theta}{1+\cos\theta}$$

the hard way

$$\frac{1+\sec\theta}{1+\cos\theta} = \sec\theta$$

$$\frac{1+\frac{1}{\cos\theta}}{1+\cos\theta} = \sec\theta$$

$$\frac{\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}}{1+\cos\theta} = \sec\theta$$

$$\frac{\frac{\cos\theta+1}{\cos\theta}}{1+\cos\theta} = \sec\theta$$

$$\frac{\cancel{\cos\theta+1}}{\cos\theta}\cdot\frac{1}{\cancel{1+\cos\theta}} = \sec\theta$$

$$\frac{1}{\cos\theta} = \sec\theta$$

$$\sec\theta = \sec\theta \checkmark$$

(7) $\tan(-x)\cot(-x) - \cos^2 x$

$-\tan x \cdot -\cot x - \cos^2 x$

$-\cancel{\tan x} \cdot \frac{-1}{\cancel{\tan x}} - \cos^2 x$

$1 - \cos^2 x$

$\boxed{\sin^2 x}$

(8) $(\csc\theta - 1)(\csc\theta + 1)$

$\csc^2\theta - \csc\theta + \csc\theta - 1$

$\csc^2\theta - 1$

$\boxed{\cot^2\theta}$

(9) $\frac{\sin x + 1 - \cos^2 x}{1 + \sin x}$

$\frac{\sin x + 1 - (1 - \sin^2 x)}{1 + \sin x}$

$\frac{\sin x + \cancel{1} - \cancel{1} + \sin^2 x}{1 + \sin x}$

$\frac{\sin x \cancel{(1 + \sin x)}}{\cancel{1 + \sin x}}$

$\boxed{\sin x}$

(10) $\frac{\sin\theta}{1 + \cos\theta} \cdot \frac{\sin\theta}{\sin\theta} + \frac{(1 + \cos\theta)}{\sin\theta} \cdot \frac{(1 + \cos\theta)}{(1 + \cos\theta)} = 2\csc\theta$

$\frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta + \sin\theta\cos\theta} = 2\csc\theta$

$\frac{1 + 1 + 2\cos\theta}{\sin\theta + \sin\theta\cos\theta} = 2\csc\theta$

$\frac{2 + 2\cos\theta}{\sin\theta + \sin\theta\cos\theta} = 2\csc\theta$

$\frac{2\cancel{(1 + \cos\theta)}}{\sin\theta\cancel{(1 + \cos\theta)}} = 2\csc\theta$

$\frac{2}{\sin\theta} = 2\csc\theta$

$2 \cdot \frac{1}{\sin\theta} = 2\csc\theta$

$2\csc\theta = 2\csc\theta$ $\square$

(14) $\cos(2\theta) = 2\cos^2\theta - 1$

$\cos(\theta+\theta) = 2\cos^2\theta - 1$

$\cos\theta\cos\theta - \sin\theta\sin\theta = 2\cos^2\theta - 1$

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - (1-\cos^2\theta) = 2\cos^2\theta - 1$

$\cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta - 1 = 2\cos^2\theta - 1$ ✓

(15) $\cos(2\theta) = 2\cos^2\theta - 1$

$+1 \qquad +1$

$\dfrac{\cos(2\theta)+1}{2} = \dfrac{2\cos^2\theta}{2}$

$\sqrt{\dfrac{\cos(2\theta)+1}{2}} = \sqrt{\cos^2\theta}$

$\pm\sqrt{\dfrac{\cos(2\theta)+1}{2}} = \cos\theta$

Let $\alpha = 2\theta$ so $\frac{\alpha}{2} = \theta$

$\pm\sqrt{\dfrac{1+\cos\alpha}{2}} = \cos\frac{\alpha}{2}$