Formulas

$$I = Prt$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$A(t) = A_0 e^{kt}$$

$$u(t) = T + (u_0 - T)e^{kt}$$

$$P(t) = \frac{c}{1 + ae^{-bt}}$$

$$FV = R\left(\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}\right)$$

$$PV = R\left(\frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right)$$

words to know

effective rate of interest

APY v. APR

annuity

half-life

Boardwork

1) Kristin invests \$100 at 6% compounded quarterly. Find the amount after 7 years.

$$A = 100\left(1+\frac{.06}{4}\right)^{(4\cdot 7)}$$

$$= \boxed{\$151.72}$$

2) Find the principal needed to get \$4,000 after 5 years at 4% compounded ~~daily~~ quarterly.

$$\frac{4000}{\left(1+\frac{.04}{4}\right)^{(4\cdot 5)}} = \frac{P\left(1+\frac{.04}{\cancel{365}\,4}\right)^{(4\cdot 5)}}{\left(1+\frac{.04}{4}\right)^{(4\cdot 5)}}$$

$$\boxed{\$3278.18 = P}$$

3) Which is more attractive to the investor:

9% compounded continuously

9.25% compounded semiannually

$$A = 100e^{.09(1)} \approx 109.41$$

$$A = 100\left(1+\frac{.0925}{2}\right)^{(2\cdot 1)} \approx 109.46$$

The 9.25% compounded semi-annually is more attractive.

4) Find the effective rate of interest for 5.3% compounded continuously.

$$A = 100e^{.053(1)} = 105.44$$

$$5.44 = 100r(1)$$

$$r = .0544 = \boxed{5.44\%}$$

5) If Todd has \$100 to invest at 10% per annum compounded monthly, how long will it take before he has \$150?

$$150 = 100\left(1+\frac{.1}{12}\right)^{12t}$$

$$1.5 = \left(1+\frac{.1}{12}\right)^{12t}$$

$$\frac{\log_{\left(1+\frac{.1}{12}\right)} 1.5}{12} = \frac{12t}{12}$$

$$\boxed{t \approx 4.08\text{yrs.}}$$

6) How long will it take for an investment to double at 6.75% compounding continuosly?

$$2 = 1e^{.0675t}$$

$$\frac{\ln 2}{.0675} = \frac{.0675t}{.0675}$$

$$\boxed{10.27\ \text{yrs} = t}$$

1) Jolene wants to purchase a new home. Suppose she invests $400 per month into a mutual fund. If the per annum rate of return is 10% compounded monthly, how much will Jolene have for a down payment after 3 years?

$$FV = 400\left(\frac{\left(1+\frac{.1}{12}\right)^{12\cdot 3}-1}{\frac{.1}{12}}\right)$$

$$= \$16,712.72$$

) Jolene has a mortgage of $150,000 for 30 years. How much will her monthy payments be if she has a loan with 5.125% APR?

$$150,000 = R\left(\frac{1-\left(1+\frac{.05125}{12}\right)^{(-12\cdot 30)}}{\frac{.05125}{12}}\right)$$

$$\boxed{R \approx \$816.74}$$

Boardwork

① Iodine 131 is a radioactive material that decays according to the function $A(t) = A_0 e^{-.087t}$. Assume a scientist has a sample of 100g of iodine 131.

a) what is the decay rate?

b) How much iodine 131 is left after 9 days?

c) when will 70 grams of iodine 131 be left?

d) what is the half-life of iodine 131?

a) $\boxed{8.7\%}$

b) $100e^{-.087(9)} = \boxed{45.70g}$

c) $70 = 100e^{-.087t}$

$.7 = e^{-.087t}$

$\frac{\ln .7}{-.087} = \frac{-.087t}{-.087}$

$t \approx \boxed{4.10 \text{ days}}$

d) $\frac{1}{2} = e^{-.087t}$

$\frac{\ln \frac{1}{2}}{-.087} = t \approx \boxed{7.97 \text{ days}}$

② the population of a city follows exponential law. If the population decreased from 900,000 to 800,000 from 2003 to 2005, what will the population be in 2010? (Leave k exact)

$A(t) = A_0 e^{kt}$

$800{,}000 = 900{,}000e^{k \cdot 2}$

$\frac{8}{9} = e^{2k}$

$\frac{\ln \frac{8}{9}}{2} = \frac{2k}{2}$

$k = \frac{\ln \frac{8}{9}}{2}$

$A(t) = 900{,}000e^{\frac{\ln \frac{8}{9}}{2}(7)}$

$= \boxed{595{,}948 \text{ people}}$

3) A bottle of water has a temperature of 61°F. It is placed in the refridgerator with a constant temperature of 38°F. After 10 min, the temperature of the bottled water is 56°F.

a) Find an equation to model the water's temperature.

b) What will the temperature of the water be after 30 min?

c) How long will it take for the water to be 41°F?

a) $u(t) = T + (u_0 - T)e^{kt}$

$56 = 38 + (61-38)e^{10k}$

$\frac{18}{23} = e^{10k}$

$\ln\frac{18}{23} = 10k$

$k = \frac{\ln\frac{18}{23}}{10}$

$\boxed{u(t) = 38 + 23e^{\frac{\ln\frac{18}{23}}{10}t}}$

b) $u(30) = 38 + 23e^{\left(\frac{\ln\frac{18}{23}}{10}\cdot 30\right)}$

$= 49.02°F$

c) $41 = 38 + 23e^{\left(\frac{\ln\frac{18}{23}}{10}t\right)}$

$\frac{3}{23} = e^{\frac{\ln\frac{18}{23}}{10}t}$

$\ln\frac{3}{23} = \frac{\ln\frac{18}{23}}{10}t$

$\boxed{t = 83.10 \text{ min}}$

④ Here is the logistic model for the population of Dallas.

$$P(t) = \frac{1{,}301{,}642}{1 + 21.602e^{-.05054t}} \quad \boxed{\begin{matrix}1900 \text{ is} \\ t=0\end{matrix}}$$

a) What is the carrying capacity?

b) What is the population in 2010?

c) When will the population be 1,000,000?

a) 1,301,642

b) $P(110) = \dfrac{1{,}301{,}642}{1 + 21.602e^{-.05054(11}}$

$= 1{,}201{,}673$

c) $1{,}000{,}000 = \dfrac{1{,}301{,}642}{1 + 21.602e^{-.050}}$

$1 + 21.602e^{-.05054t} = \dfrac{1301642}{1000000}$

$21.602e^{-.05054t} = 1.301642$

$e^{-.05054t} = \dfrac{.301642}{21.602}$

$\ln \dfrac{.301642}{21.602} = -.05054t$

$\boxed{t \approx 84.5 \text{ yrs. after } 1900 \text{ or } 1984}$

Population of Lincoln

Historical population		
Census	Pop.	%±
1870	2,441	—
1880	13,003	432.7%
1890	55,164	324.2%
1900	40,169	−27.2%
1910	43,973	9.5%
1920	54,948	25.0%
1930	75,933	38.2%
1940	81,984	8.0%
1950	98,884	20.6%
1960	128,521	30.0%
1970	149,518	16.3%
1980	171,932	15.0%
1990	191,972	11.7%
2000	225,581	17.5%
2010	258,379	14.5%
Est. 2012	265,404	2.7%

U.S. Decennial Census[17]
2012 Estimate[18]

Population of Omaha

Historical population		
Census	Pop.	%±
1860	1,883	—
1870	16,083	754.1%
1880	30,518	89.8%
1890	140,452	360.2%
1900	102,555	−27.0%
1910	124,096	21.0%
1920	191,061	54.0%
1930	214,006	12.0%
1940	223,844	4.6%
1950	251,117	12.2%
1960	301,598	20.1%
1970	346,929	15.0%
1980	313,939	−9.5%
1990	335,795	7.0%
2000	390,007	16.1%
2010	408,958	4.9%
Est. 2012	421,570	3.1%

source:[116][117]

Find a model for each.