

① Find the first 5 terms of each sequence.

$$\begin{aligned} \text{a)} \quad a_n &= -2a_{n-1} + 4 \\ a_1 &= -3 \end{aligned}$$

$$\text{b)} \quad b_n = \frac{\pi^{n-1}}{2n}$$

① a)

$$\begin{aligned} a_1 &= -3 \\ a_2 &= -2(-3) + 4 = 10 \\ a_3 &= -2(10) + 4 = -16 \\ a_4 &= -2(-16) + 4 = 36 \\ a_5 &= -2(36) + 4 = -68 \end{aligned}$$

b)

$$\begin{aligned} b_1 &= \frac{\pi^0}{2(1)} = \frac{1}{2} \\ b_2 &= \frac{\pi^1}{2(2)} = \frac{\pi}{4} \\ b_3 &= \frac{\pi^2}{2(3)} = \frac{\pi^2}{6} \\ b_4 &= \frac{\pi^3}{2(4)} = \frac{\pi^3}{8} \\ b_5 &= \frac{\pi^4}{2(5)} = \frac{\pi^4}{10} \end{aligned}$$

② Given that the pattern continues, find the n^{th} term of the sequence (explicit form).

a) $1, -8, 27, -64, 125, \dots$

b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$

② a) $a_n = (-1)^{n-1} n^3$

b) $a_n = \frac{n}{n+1}$

③

a) Write out the sum.

$$\sum_{k=1}^n \frac{1}{e^{n+k}}$$

b) Express the sum using summation notation.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots + \frac{1}{100}$$

③ a) $\frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \dots + \frac{1}{e^{n+1}}$

b) $\sum_{k=1}^{50} \frac{1}{2k}$

④ Find the sum.

a)
$$\sum_{k=1}^{12} 6k^2 + k$$

b)
$$\sum_{k=4}^9 2k^3$$

④ a)
$$6 \sum_{k=1}^{12} k^2 + \sum_{k=1}^{12} k$$

$$6 \left[\frac{12(13)(2(12)+1)}{6} \right] + \frac{12(13)}{2}$$

$$3900 + 78$$

$$\boxed{3978}$$

b)
$$2 \sum_{k=1}^9 k^3 - \sum_{k=1}^3 k^3$$

$$2 \sum_{k=1}^9 k^3 - 2 \sum_{k=1}^3 k^3$$

$$2 \left[\frac{9(10)}{2} \right]^2 - 2 \left[\frac{3(4)}{2} \right]^2$$

$$4050 - 72$$

$$\boxed{3978}$$

$$\textcircled{5} \text{ a) } a_1 = 2 - 7(1)$$

$$\boxed{a_1 = -5}$$

$$d = a_n - a_{n-1}$$

$$d = 2 - 7n - (2 - 7(n-1))$$

$$d = 2 - 7n - (2 - 7n + 7)$$

$$d = 2 - 7n - (9 - 7n)$$

$$d = 2 - 7n - 9 + 7n$$

$$\boxed{d = -7}$$

$$\text{b) } b_1 = 4^{3(1)}$$

$$\boxed{b_1 = 64}$$

$$r = \frac{a_n}{a_{n-1}}$$

$$r = \frac{4^{3n}}{4^{3(n-1)}} = \frac{4^{3n}}{4^{3n-3}}$$

$$r = 4^3$$

$$\boxed{r = 64}$$

$\textcircled{5}$ a) Show that the sequence is arithmetic.

$$a_n = 2 - 7n$$

b) Show that the sequence is geometric

$$b_n = 4^{3n}$$

* Show means find the first term and prove common difference

⑥ The 12th term of an arithmetic sequence is 91 and the 31st term is 243

a) Find a recursive formula for the sequence.

b) Find an explicit formula for the sequence. (n^{th} term)

c) Find the 501st term.

⑥

$$a_{12} = d(12) + a_1 - d = 91$$

$$a_{31} = d(31) + a_1 - d = 243$$

$$11d + a_1 = 91$$

$$30d + a_1 = 243$$

$$\frac{19d}{19} = \frac{152}{19}$$

$$d = 8$$

$$11(8) + a_1 = 91$$

$$88 + a_1 = 91$$

$$a_1 = 3$$

$$a) \begin{cases} a_1 = 3 \\ a_n = a_{n-1} + 8 \end{cases}$$

$$b) a_n = 8n - 5$$

$$c) a_{501} = 8(501) - 5$$

$$a_{501} = 4003$$

⑦ Find each sum.

a) $\sum_{n=1}^{17} (3n+4)$

b) $-5.5 + (-5) + (-4.5) + \dots + 19.5$

⑦ a) arithmetic $\Rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

$$S_{17} = \frac{17}{2}(7 + 55)$$

$$S_{17} = \boxed{527}$$

b) arithmetic $\Rightarrow S_n = \frac{n}{2}(a_1 + a_n)$

need to find n first

$$a_n = .5n - 6$$

$$19.5 = .5n - 6$$

$$25.5 = .5n$$

$$n = 51$$

$$S_{51} = \frac{51}{2}(-5.5 + 1)$$

$$S_{51} = \boxed{357}$$

⑧ The following is a geometric sequen

$$5, \frac{25}{2}, \frac{125}{4}, \frac{625}{8}, \dots$$

a) Find a recursive formula for the sequ

b) Find an explicit formula for the sequ
(nth term)

c) Find the 8th term.

⑧

a) $a_1 = 5$

$$a_n = \frac{5}{2} a_{n-1}$$

b) $a_n = 5 \left(\frac{5}{2}\right)^{n-1}$

c) $a_8 = 5 \left(\frac{5}{2}\right)^{8-1} = 5 \cdot \frac{5^7}{2^7} = \boxed{\frac{390625}{128}}$

⑧ Find each sum.

a) $\sum_{k=1}^{10} 7 \left(\frac{1}{4}\right)^{k-1}$

b) $8 + 16 + 32 + 64 + \dots + 32,768$

⑨

a) $S_n = a_1 \frac{1-r^n}{1-r}$

$$S_{10} = 7 \cdot \frac{1 - \left(\frac{1}{4}\right)^{10}}{1 - \left(\frac{1}{4}\right)} = \frac{7 \left(1 - \frac{1}{1048576}\right)}{\frac{3}{4}} = \frac{7 \left(\frac{1048575}{1048576}\right)}{\frac{3}{4}}$$

$$= \frac{\frac{7340025}{1048576}}{\frac{3}{4}} = \frac{2446675}{1048576} \cdot \frac{4}{3} = \frac{2446675}{262144}$$

b) $S_n = a_1 \frac{1-r^n}{1-r}$

$$a_n = 8(2)^{n-1}$$

$$32768 = 8(2)^{n-1}$$

$$4096 = 2^{n-1}$$

$$\log_2 4096 = n-1$$

$$12 = n-1$$

$$13 = n$$

$$S_{13} = 8 \cdot \frac{1 - 2^{13}}{1 - 2}$$

$$= \frac{2446675}{262144}$$

10 Determine whether the sequence is arithmetic, geometric, or neither. If it is arithmetic, find the common difference. If it is geometric, find the common r .

a) $a_n = 3\left(\frac{1}{2^n}\right)$

b) $b_n = -6n^3$

c) $c_n = 16 - n$

10 a) geometric $r = \frac{1}{2}$

$$\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \dots$$

b) neither

c) arithmetic $d = -1$

$$15, 14, 13, \dots$$

⑪ Determine whether each infinite geometric series converges or diverges. If it converges, find its sum.

a) $50 + 10 + 2 + \frac{2}{5} + \dots$

b) $\frac{1}{7} + \frac{-1}{14} + \frac{1}{28} + \frac{-1}{56} + \dots$

c) $\sum_{k=1}^{\infty} \left(\frac{5}{3}\right)^{k-1}$

⑪ a) $r = \frac{1}{5} \Rightarrow$ converges

$$S = \frac{a_1}{1-r} = \frac{50}{1-\frac{1}{5}} = \boxed{62.5}$$

b) $r = \frac{-1}{2} \Rightarrow$ converges

$$S = \frac{a_1}{1-r} = \frac{\frac{1}{7}}{1-\frac{-1}{2}} = \frac{\frac{1}{7}}{\frac{3}{2}} = \frac{1}{7} \cdot \frac{2}{3} = \boxed{\frac{2}{21}}$$

c) $r = \frac{5}{3} \Rightarrow$ diverges