1. $x^2 + y^2 - 2x + 4y - 4 = 0$

a) Identify the conic (circle, parabola, ellipse, or hyperbola).
b) If it is a circle, identify the center and radius.
   If it is a parabola, identify the vertex, focus, directrix, and points that define the latus rectum.
   If it is an ellipse, identify the center, vertices, and foci.
   If it is a hyperbola, identify the center, transverse axis, vertices, foci, and asymptotes.
c) Graph the conic.

2. $2y^2 - x^2 + 2x + 8y + 3 = 0$

a) Identify the conic (circle, parabola, ellipse, or hyperbola).
b) If it is a circle, identify the center and radius.
   If it is a parabola, identify the vertex, focus, directrix, and points that define the latus rectum.
   If it is an ellipse, identify the center, vertices, and foci.
   If it is a hyperbola, identify the center, transverse axis, vertices, foci, and asymptotes.
c) Graph the conic.

3. $4x^2 + y^2 + 8x - 4y + 4 = 0$

a) Identify the conic (circle, parabola, ellipse, or hyperbola).
b) If it is a circle, identify the center and radius.
   If it is a parabola, identify the vertex, focus, directrix, and points that define the latus rectum.
   If it is an ellipse, identify the center, vertices, and foci.
   If it is a hyperbola, identify the center, transverse axis, vertices, foci, and asymptotes.
c) Graph the conic.

4. $4x^2 - 16x + 16y + 32 = 0$

a) Identify the conic (circle, parabola, ellipse, or hyperbola).
b) If it is a circle, identify the center and radius.
   If it is a parabola, identify the vertex, focus, directrix, and points that define the latus rectum.
   If it is an ellipse, identify the center, vertices, and foci.
   If it is a hyperbola, identify the center, transverse axis, vertices, foci, and asymptotes.
c) Graph the conic.

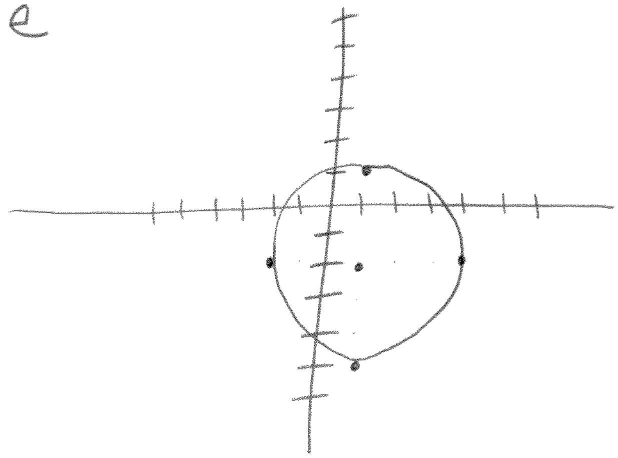
① $x^2+y^2-2x+4y-4=0$

$(x^2-2x+\underline{1})+(y^2+4y+\underline{4})-4-\underline{1}-\underline{4}=0$

$(x-1)^2+(y+2)^2=9 \Rightarrow$ circle

center: $(1,-2)$

radius: 3



② $2y^2-x^2+2x+8y+3=0$

$2y^2+8y+\_\_\_-x^2+2x+\_\_\_+3-\_\_\_-\_\_\_=0$

$2(y^2+4y+\underline{4})-(x^2-2x+\underline{1})+3-\underline{8}+\underline{1}=0$

$\dfrac{2(y+2)^2}{4}-\dfrac{(x-1)^2}{4}=\dfrac{4}{4}$

$\dfrac{(y+2)^2}{2}-\dfrac{(x-1)^2}{4}=1 \Rightarrow$ hyperbola

$a=\sqrt{2}\approx 1.4 \quad b=2$

$b^2=c^2-a^2$

$4=c^2-2$

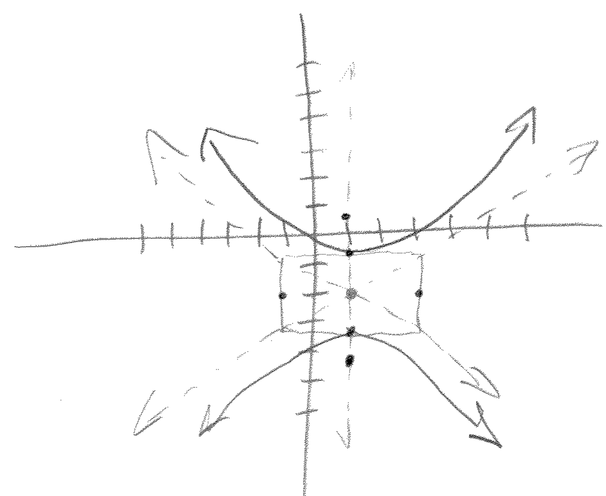
$c=\sqrt{6}\approx 2.4$

center: $(1,-2)$

transverse axis: $x=1$

vertices: $(1,-2+\sqrt{2})$
$(1,-2-\sqrt{2})$

foci: $(1,-2+\sqrt{6})$
$(1,-2-\sqrt{6})$



asymptotes: $y+2=\pm\frac{\sqrt{2}}{2}(x-1)$

③ $4x^2+y^2+8x-4y+4=0$

$4x^2+8x+\_\_\_+y^2-4y+\_\_\_+4-\_\_\_-\_\_\_=0$

$4(x^2+2x+\underline{1})+(y^2-4y+\underline{4})+4-\underline{4}-\underline{4}=0$

$\frac{4(x+1)^2}{4}+\frac{(y-2)^2}{4}=\frac{4}{4}$

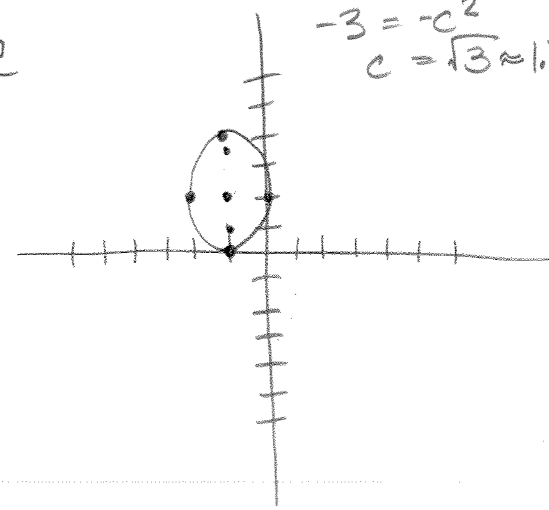
$a=2 \quad b^2=a^2-c^2$

$b=1 \quad 1^2=2^2-c^2$

$-3=-c^2$

$c=\sqrt{3}\approx 1.7$

$\frac{(x+1)^2}{1}+\frac{(y-2)^2}{4}=1 \Rightarrow$ ellipse



center: $(-1,2)$

vertices: $(-1,0)$

$(-1,4)$

foci: $(-1, 2+\sqrt{3})$

$(-1, 2-\sqrt{3})$

④ $4x^2-16x+16y+32=0$

$4(x^2-4x+\underline{4})+16y+32-\underline{16}=0$

$4(x-2)^2+16y+16=0$

$4(x-2)^2=-16y-16$

$\frac{4(x-2)^2}{4}=\frac{-16(y+1)}{4}$

$(x-2)^2=-4(y+1) \Rightarrow$ parabola

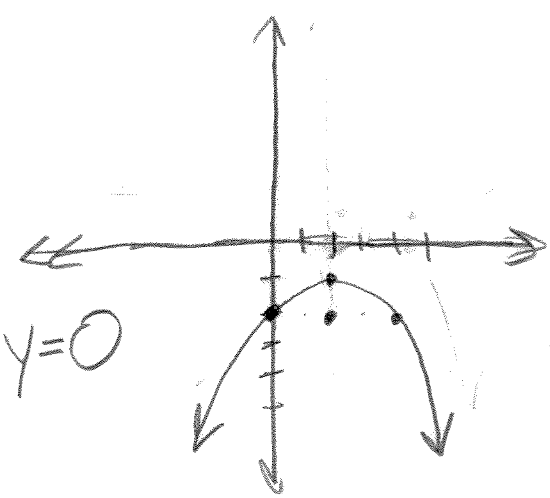
vertex: $(2,-1)$ $\quad a=-1$

focus: $(2,-2)$

directrix: $y=2$

points that define latus rectum: $y=0$

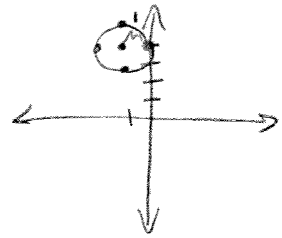
$(0,-2)$, $(4,-2)$



Circle: center $(-1,4)$ + tangent to the y-axis

equation: $(x-h)^2+(y-k)^2=r^2$

$$\boxed{(x+1)^2+(y-4)^2=1}$$



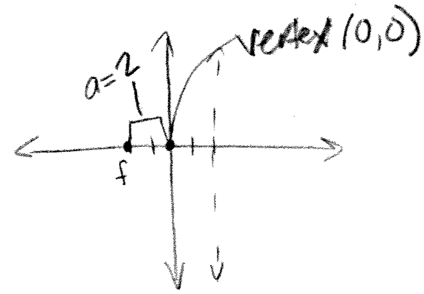
Parabola: focus $(-2,0)$; directrix $x=2$

equation: $(y-k)^2=4a(x-h)$

$y^2=4ax$

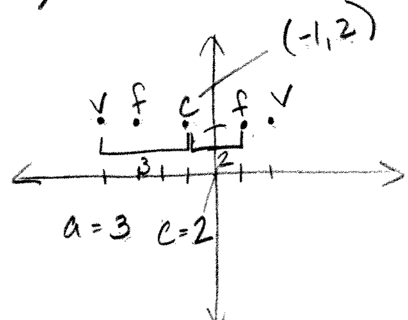
$y^2=4(-2)x$

$$\boxed{y^2=-8x}$$



Ellipse: foci $(1,2)$ + $(-3,2)$; vertex $(-4,2)$

equation: $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$



$b^2=a^2-c^2$

$b^2=3^2-2^2$

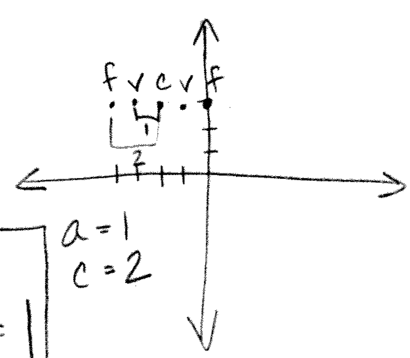
$b^2=5$

$b=\sqrt{5}$

$$\boxed{\frac{(x+1)^2}{9}+\frac{(y-2)^2}{5}=1}$$

Hyperbola: center $(-2,3)$; focus $(-4,3)$; vertex $(-3,3)$

equation: $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$



$b^2=c^2-a^2$

$b^2=2^2-1^2$

$b^2=3$

$b=\sqrt{3}$

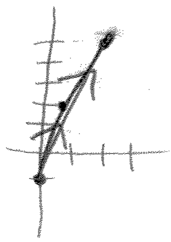
$$\boxed{\frac{(x+2)^2}{1}-\frac{(y-3)^2}{3}=1}$$

Boardwork.

Graph. Find the rectangular equation of the curve.

① $x = \frac{1}{2}t \quad y = 3t - 1 \quad 0 \le t \le 2$

t	x	y
0	0	-1
1	.5	2
2	1	5



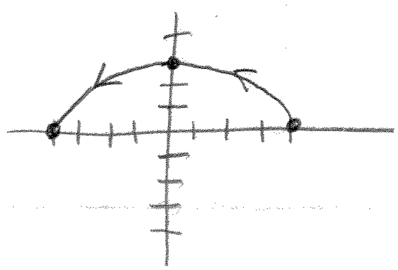
$t = 2x$

$y = 3(2x) - 1$

$\boxed{y = 6x - 1}$

② $x = 4\cos t \quad y = 3\sin t \quad 0 \le t \le \pi$

t	x	y
0	4	0
$\frac{\pi}{2}$	0	3
π	-4	0



$\cos t = \frac{x}{4} \quad \sin t = \frac{y}{3}$

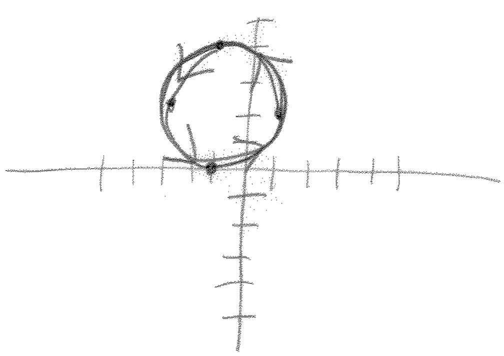
$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\boxed{\frac{x^2}{16} + \frac{y^2}{9} = 1}$

③ $x = 2\cos t - 1 \quad y = 2\sin t + 2 \quad 0 \le t \le 2\pi$

t	x	y
0	1	2
$\frac{\pi}{2}$	-1	4
π	-3	2
$\frac{3\pi}{2}$	-1	0
2π	1	2



$\cos t = \frac{x+1}{2} \quad \sin t = \frac{y-2}{2}$

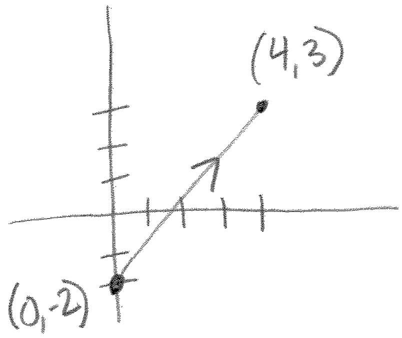
$\left(\frac{x+1}{2}\right)^2 + \left(\frac{y-2}{2}\right)^2 = 1$

$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{4} = 1$

$\boxed{(x+1)^2 + (y-2)^2 = 4}$

Find parametric equations

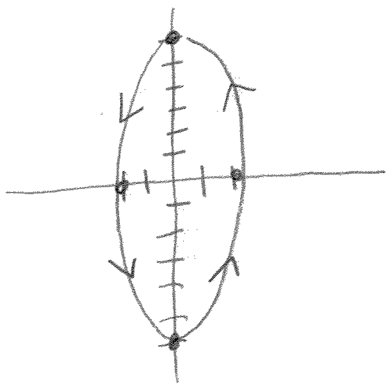
1)



t	x	y
0	0	-2
1	4	3

$$\boxed{\begin{aligned} x(t) &= 4t \\ y(t) &= -2 + 5t \\ 0 \le\ & t \le 1 \end{aligned}}$$

2)



$$\boxed{\begin{aligned} x(t) &= 2\cos t \\ y(t) &= 6\sin t \\ 0 \le\ & t \le 2\pi \end{aligned}}$$

Boardwork

Ex. Suppose Maddie threw a ball from a height of 5 m with an initial velocity of 20 m/s at an angle of 15° to the horizontal.

a) Find parametric equations that describe the position of the ball as a function of time.

$x = (20\cos 15^\circ)t$

$y = -4.9t^2 + (20\sin 15^\circ)t +$

b) How long is the ball in the air?

$0 = -4.9t^2 + 5.176t + 5$

$$t = \frac{-5.176 \pm \sqrt{(5.176)^2 - 4(-4.9)(5)}}{2(-4.9)}$$

$$t \approx \frac{-5.176 \pm 11.171}{-9.8}$$

$t \approx 1.67$ sec $\quad t \approx \cancel{}$

c) When is the ball at its maximum height? Determine the maximum height of the ball.

$$t = \frac{-5.176}{2(-4.9)} \approx .528\text{ sec}$$

$y = -4.9(.528)^2 + 5.176(.528) + 5$

$y = \boxed{6.367\text{ m}}$

d) Determine the horizontal distance the ball traveled.

$x = 20\cos 15^\circ (1.67)$

$x \approx \boxed{32.26\text{ m}}$