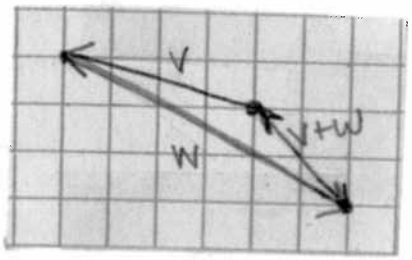
Vectors Review Boardwork

Let $v = -4i + j$ and $w = 6i - 3j$

(1) Graph $v+w$.

(2) Find $2v - 3w$.

(1)



(2) $2(-4i + j) - 3(6i - 3j)$

$-8i + 2j - 18i + 9j$

$\boxed{-26i + 11j}$

(3) Find $\|2w - v\|$

(3) $2w - v = 2(6i - 3j) - (-4i + j)$

$= 12i - 6j + 4i - j$

$= 16i - 7j$

$\|2w - v\| = \sqrt{16^2 + (-7)^2}$

$= \boxed{\sqrt{305}}$

(4) Find the unit vector in the direction of w

(4) $u = \frac{w}{\|w\|}$

$u = \frac{6i - 3j}{\sqrt{36+9}} = \frac{6i - 3j}{\sqrt{45}} = \frac{6i}{3\sqrt{5}} + \frac{-3j}{3\sqrt{5}}$

$= \frac{2}{\sqrt{5}}i - \frac{1}{\sqrt{5}}j$

(5) Decompose vector v into two vectors v_1 and v_2 such that v_1 is parallel to w and v_2 is orthogonal to w

(5) $v_1 = \frac{v \cdot w}{\|w\|^2} w = \frac{-4(6) + 1(-3)}{\left(\sqrt{6^2 + (-3)^2}\right)^2}(6i - 3j)$

$= \frac{-27}{45}(6i - 3j) = \frac{-3}{5}(6i - 3j)$

$= \boxed{\frac{-18}{5}i + \frac{9}{5}j}$

$v_2 = -4i + j - \left(\frac{-18}{5}i + \frac{9}{5}j\right)$

$= \boxed{\frac{-2}{5}i - \frac{4}{5}j}$

(6) Find the angle between v and w.

$$\cos\theta = \frac{v \cdot w}{\|v\|\|w\|} = \frac{-4(6)+1(-3)}{\sqrt{(-4)^2+1^2}\cdot\sqrt{6^2+(-3)^2}}$$

$$\cos\theta = \frac{-27}{\sqrt{17}\cdot\sqrt{45}} = \frac{-27}{\sqrt{765}}$$

$$\boxed{\theta = 167.47^\circ}$$

Are the vectors parallel, orthogonal or neither?

(7) $v = 9i+3j \quad w = -3i-j$

(8) $v = 4i-6j \quad w = -3i-5j$

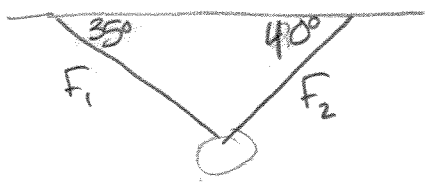
(9) $v = 6i-12j \quad w = 8i+4j$

(7) Since $v = -3w$, the vectors are parallel

(9) Since $v \cdot w = 0$, the vectors are orthogonal

(8) Since $v \neq \alpha w$ and $v \cdot w \neq 0$, the vectors are neither.

(10) A chandelier of 100 lbs hangs from the ceiling suspended by two ropes. What is the tension in each rope?



(10) $F_1 + F_2 + F_3 = 0$

$$\|F_1\|(\cos 145^\circ i + \sin 145^\circ j) + \|F_2\|(\cos 20^\circ i + \sin 20^\circ j) - 100 = 0$$

$$\begin{cases} \|F_1\|\cos 145^\circ + \|F_2\|\cos 40^\circ = 0 \\ \|F_1\|\sin 145^\circ + \|F_2\|\sin 40^\circ - 100 = 0 \end{cases}$$

$$\|F_1\| = \frac{-\|F_2\|\cos 40^\circ}{\cos 145^\circ}$$

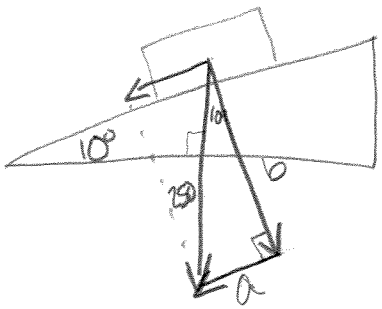
$$-\frac{\|F_2\|\cos 40^\circ}{\cos 145^\circ}\sin 145^\circ + \|F_2\|\sin 40^\circ = 100$$

$$\|F_2\|\left(\frac{\cos 40^\circ \sin 145^\circ}{\cos 145^\circ} + \sin 40^\circ\right) = 100$$

$$\|F_2\| = \boxed{84.8 \text{ lbs}}$$

$$\|F_1\| = \frac{-84.8\cos 40^\circ}{\cos 145^\circ} = \boxed{53.2 \text{ lbs}}$$

(11) A box of 250lbs in on an incline with a slope of 10°. What is the force required to keep the box from sliding down? What is the force prependicular to the incline?



$\sin 10° = \frac{a}{250}$

$a = 250 \sin 10° \approx$ 43.41 lbs

to keep box from sliding dow

$\cos 10° = \frac{b}{250}$

$b = 250 \cos 10° \approx$ 246.201

force perpendic to incline

(12) You are pulling an object with a force of 60lbs in the direction 20° to the horizontal. How much work is done in moving the object 5ft?

$W = F \cdot \overrightarrow{AB}$

$W = 60(\cos 20° i + \sin 20° j) \cdot 5$

$W = (56.38i + 20.52j) \cdot 5i$

$W =$ 281.9 ft-lbs