① $\tan\theta + \cot\theta = \sec\theta\csc\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} =$$

$$\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} =$$

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} =$$

$$\frac{1}{\sin\theta\cos\theta} =$$

$$\frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} =$$

$$\csc\theta \cdot \sec\theta = \sec\theta\csc\theta \checkmark$$

② $$\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$\frac{\cos\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} =$$

$$\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} =$$

$$\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} =$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta} \checkmark$$

(3) $\tan\left(\theta - \frac{3\pi}{2}\right) = -\cot\theta$

$$\frac{\sin\left(\theta - \frac{3\pi}{2}\right)}{\cos\left(\theta - \frac{3\pi}{2}\right)} =$$

$$\frac{\sin\theta\cos\frac{3\pi}{2} - \cos\theta\sin\frac{3\pi}{2}}{\cos\theta\cos\frac{3\pi}{2} + \sin\theta\sin\frac{3\pi}{2}} =$$

$$\frac{\sin\theta \cdot 0 - \cos\theta \cdot (-1)}{\cos\theta \cdot 0 + \sin\theta \cdot (-1)} =$$

$$\frac{\cos\theta}{-\sin\theta} =$$

$$-\cot\theta = -\cot\theta \checkmark$$

(4) $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta$

$$\frac{\sin\theta}{1+\cos\theta} \cdot \frac{\sin\theta}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} = 2\csc\theta$$

$$\frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta} = 2\csc\theta$$

$$\frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta} = 2\csc\theta$$

$$\frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} = 2\csc\theta$$

$$\frac{2}{\sin\theta} = 2\csc\theta$$

$$2\csc\theta = 2\csc\theta$$

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$$\frac{\tan v + \cot v}{\sec v \csc v} = 1$$

$$\frac{\dfrac{\sin v}{\cos v} + \dfrac{\cos v}{\sin v}}{\dfrac{1}{\cos v} \cdot \dfrac{1}{\sin v}} = 1$$

$$\frac{\dfrac{\sin^2 v + \cos^2 v}{\cos v \sin v}}{\dfrac{1}{\cos v \sin v}} = 1$$

$$\frac{\dfrac{1}{\cos v \sin v}}{\dfrac{1}{\cos v \sin v}} = 1$$

$$1 = 1 \checkmark$$

(A) Evaluate $\cos 75^\circ$

(B) Evaluate $\tan \frac{\pi}{12}$

(C) Given $\sin u = \frac{-3}{5}$ $\pi < u < \frac{3\pi}{2}$ and $\cos v = \frac{12}{13}$ $0 < v < \frac{\pi}{2}$

a) Find $\sin(u - v)$

b) Find $\sin \frac{u}{2}$

(D) Evaluate $\cos 112.5^\circ$

(E) If $\cos\theta = \frac{1}{4}$, θ in Q4, find exact value of $\tan\left(\theta + \frac{\pi}{4}\right)$.

(A) Evaluate $\cos 75^\circ$

$$\cos(30^\circ + 45^\circ) = \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

(B) Evaluate $\tan \frac{\pi}{12}$.

$$\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \cdot \tan \frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$$

$$= \frac{(\sqrt{3} - 1)}{(1 + \sqrt{3})} \cdot \frac{(1 - \sqrt{3})}{(1 - \sqrt{3})} = \frac{\sqrt{3} - 1 - 3 + \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - 3}$$

$$= \frac{-4 + 2\sqrt{3}}{-2} = \boxed{2 - \sqrt{3}}$$

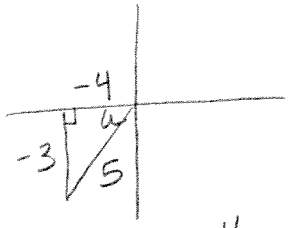
-OR-

$$\boxed{\sqrt{7 - 4\sqrt{3}}}$$

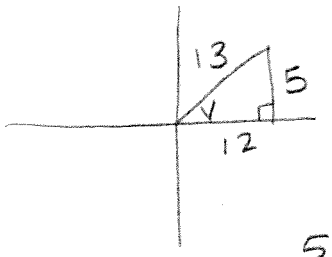
(C) Given $\sin u = \frac{-3}{5}$ $\pi < u < \frac{3\pi}{2}$ and

$\cos v = \frac{12}{13}$ $0 < v < \frac{\pi}{2}$.

a) Find $\sin(u-v)$.



$\cos u = \frac{-4}{5}$



$\sin v = \frac{5}{13}$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$= \frac{-3}{5} \cdot \frac{12}{13} - \frac{-4}{5} \cdot \frac{5}{13}$$

$$= \frac{-36}{65} + \frac{20}{65}$$

$$= \boxed{\frac{-16}{65}}$$

b) Find $\sin \frac{u}{2}$.

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1-\cos u}{2}}$$

$$= \sqrt{\frac{1-\frac{-4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}}$$

$$= \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

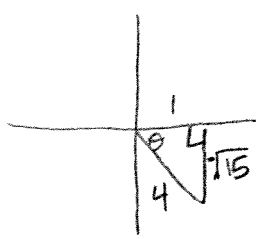
(D) Evaluate $\cos 112.5°$.

$$\cos\left(\frac{225°}{2}\right) = \pm\sqrt{\frac{1+\cos 225°}{2}}$$

$$= -\sqrt{\frac{1+\frac{-\sqrt{2}}{2}}{2}\cdot\frac{2}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \boxed{-\frac{\sqrt{2-\sqrt{2}}}{2}}$$

(E) If $\cos\theta = \frac{1}{4}$, θ in Q4, find the exact value of $\tan\left(\theta + \frac{\pi}{4}\right)$.



$$\tan\theta = \frac{-\sqrt{15}}{1}$$

$$\tan\left(\theta + \frac{\pi}{4}\right) = \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta\tan\frac{\pi}{4}}$$

$$= \frac{-\sqrt{15}+1}{1 - -\sqrt{15}\cdot 1} = \frac{(-\sqrt{15}+1)}{(1+\sqrt{15})}\cdot\frac{(1-\sqrt{15})}{(1-\sqrt{15})}$$

$$= \frac{-\sqrt{15}+15+1-\sqrt{15}}{1+\sqrt{15}-\sqrt{15}-15} = \frac{16-2\sqrt{15}}{-14}$$

$$= \boxed{\frac{8-\sqrt{15}}{-7}}$$

Unit Exam Review (2/17)

1. $\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = 1/2$

$\theta = \pi/6 + 2k\pi$ & $\theta = 5\pi/6 + 2k\pi$

2. $\sin x + \sqrt{2} = -\sin x$

$+\sin x \qquad +\sin x$

$2\sin x + \sqrt{2} = 0$

$\frac{2\sin x}{2} = \frac{-\sqrt{2}}{2}$

$\sin x = -\sqrt{2}/2$

$x = 5\pi/4 + 2k\pi$ and $x = 7\pi/4 + 2k\pi$

3. $5\cos\theta - \sqrt{2} = 3\cos\theta$

$2\cos\theta - \sqrt{2} = 0$

$2\cos\theta = \sqrt{2}$

$\cos\theta = \sqrt{2}/2$

$\theta = \pi/4 + 2k\pi$ and $\theta = 7\pi/4 + 2k\pi$

4. $\tan^2\alpha - 3 = 0$

$\sqrt{\tan^2\alpha} = \sqrt{3}$

$\tan\alpha = \pm\sqrt{3}$

$\alpha = \pi/3 + k\pi$ and $\alpha = 2\pi/3 + k\pi$

$(0 \le \theta \le 2\pi)$!

5. $2\sin^2\theta + 1 = 3\sin\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ $\qquad$ $\sin\theta - 1 = 0$

$2\sin\theta = 1$ $\qquad$ $\sin\theta = 1$

$\sin\theta = 1/2$

$\theta = \pi/2 + 2\pi k$

$\theta = \pi/6 + 2k\pi$

$\theta = 5\pi/6 + 2k\pi$

$\theta = \{\pi/6, 5\pi/6, \pi/2\}$

6. $\sec^2 x - 3\sec x = -2$

$\sec^2 x - 3\sec x + 2 = 0$

$(\sec x - 2)(\sec x - 1) = 0$

$\sec x - 2 = 0$ $\qquad$ $\sec x - 1 = 0$

$\sec x = 2$ $\qquad$ $\sec x = 1$

$x = \{\pi/3, 5\pi/3, 0\}$

7. $2\cos(3t) - 1 = 0$

$2\cos(3t) = 1$

$\cos(3t) = 1/2$

$3t = \pi/3 + 2k\pi$

$3t = 5\pi/3 + 2k\pi$

$t = \pi/9 + \frac{2}{9}k\pi$

$t = 5\pi/9 + \frac{2}{9}k\pi$

8. $\sec^2 x - 2\tan x = 4$

$1 + \tan^2 x - 2\tan x - 4 = 0$

$\tan^2 x - 2\tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

$\tan x - 3 = 0 \qquad \tan x + 1 = 0$

$\tan x = 3 \qquad \tan x = -1$

$x = \tan^{-1}(3) \qquad x = -\pi/4$

$2x^2 + x - 1$

$(2x)(x)$

9. $\cos(2\theta) + \cos\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$2\cos\theta - 1 = 0 \qquad \cos\theta + 1 = 0$

$\frac{2\cos\theta}{2} = \frac{1}{2} \qquad \cos\theta = -1$

$\cos\theta = (1/2) \qquad \cos\theta = -1$

$\theta = \{\pi/3, 5\pi/3, \pi\}$

10. $4\csc^2 x - 2 = 0$

$4\csc^2 x = 2$

$\sqrt{\csc^2 x} = \sqrt{1/2}$

$\csc x = \frac{1}{\sqrt{2}}$

$\csc x = \sqrt{2}/2$

$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

10. $\sin^2 x - 5\cos x = 5$

11. $\cos\theta = -.68$