Adv Algebra – Ch 8 and 9 Cumulative Review

Name KEY Period

For each of the following problems, ***show work or receive no credit.***

1. A new car that sells for $25,000 depreciates 15% each year.

   a) Write a model for the value *v* of the car after *t* years.

   1a. $v(t) = 25{,}000(1-.15)^t$

   $v(t) = 25{,}000(.85)^t$ OR

   b) What is the value of the car after 10 years?

   1b. $4,921.86

   $v(20) = 25{,}000(.85)^{20}$

2. If you deposit a principal amount of $1000 in an account that is compounded continuously at an annual rate of 3%, how much money would you have after 20 years?

   2. $1,822.12

   $A = Pe^{rt} \rightarrow 1000e^{(.03\times 20)}$

3. Write an exponential function of the form $y = ab^x$ that contains the points $(-2, 16)$ and $\left(3, \frac{1}{2}\right)$.

   3. $y = 4\left(\frac{1}{2}\right)^x$

   $y = a\cdot b^x$

   $16 = a\cdot b^{-2}$

   $b^2 \cdot 16 = \frac{a}{b^2}\cdot b^2$

   $a = 16b^2$

   $y = a\cdot b^x$

   $y = 16b^2\cdot b^x$

   $\frac{1}{2} = 16b^2\cdot b^3$

   $\frac{\frac{1}{2}}{16} = \frac{16b^5}{16}$

   $\frac{1}{2}\div 16$

   $\frac{1}{2}\cdot\frac{1}{16}$

   $\sqrt[5]{\frac{1}{32}} = \sqrt[5]{b^5}$

   $\frac{\sqrt[5]{1}}{\sqrt[5]{32}} = b$

   $\frac{1}{2} = b$

   $a = 16b^2$

   $= 16\left(\frac{1}{2}\right)^2$

   $= 16\cdot\frac{1}{4}$

   $a = 4$

4. Write $\log_2 8 = 3$ in exponential form.

   4. $2^3 = 8$

   bump

   $2^3 = 8$

5. Write the following as a single logarithm:
   $2\log_2 x - 3\log_2 y + 5\log_2 z$

   5. $\log_2 \frac{x^2 z^5}{y^3}$

   $\log_2 x^2 - \log_2 y^3 + \log_2 z^5$

   $\log_2 \frac{x^2}{y^3} + \log_2 z^5 = \log_2 \frac{x^2}{y^3}\cdot\frac{z^5}{1}$

6. Expand the following logarithm: $\log \frac{z\sqrt[3]{y}}{x^2}$

   6. $\log z + \frac{1}{3}\log y - 2\log x$

   $\log z\cdot y^{\frac{1}{3}} - \log x^2$

   $\log z + \log y^{\left(\frac{1}{3}\right)} - \log x^{(2)}$

   $\log z + \frac{1}{3}\log y - 2\log x$

7. Evaluate the following logarithm: $\log_3 \frac{1}{81}$

7. -4

$3^x = \frac{1}{81}$

$\frac{1}{3^{-x}} = \frac{1}{3^4}$

$-x = 4$

$x = -4$

For questions 8 – 11, solve the equation. Round to two decimal places if necessary.

8. $\log_6 16x = 5$

8. $x = 486$

Rewrite in Exponential form

$\frac{6^5}{16} = \frac{16x}{16}$

$x = \frac{6^5}{16} = 486$

9. $9^{2x-3} + 4 = 21$

9. $x \approx 2.14$

$-4 \quad -4$

$9^{2x-3} = 17$

Rewrite as a log

$\log_9 17 = 2x - 3$

$\log_9 17 + 3 = 2x$

$\frac{(\log_9 17) + 3}{2} = x$

OR

$\frac{\frac{\log 17}{\log 9} + 3}{2}$

10. $3e^{x+1} - 2 = 10$

10. $x \approx .39$

$+2 \quad +2$

$\frac{3e^{x+1}}{3} = \frac{12}{3}$

$\ln e^{x+1} = \ln 4$

$\ln e^{(x+1)} = \ln 4$

$x + 1 (\ln e) = \ln 4$

$x + 1 = \ln 4 - 1$

$x = (\ln 4) - 1$

$x \approx .39$

11. $1 - (2 \ln x) = -4$

11. $x \approx 12.18$

$-1 \quad -1$

$\frac{-2 \ln x}{-2} = \frac{-5}{-2}$

$\ln x = \frac{5}{2}$

$\ln \to \log_e$

$\log_e x = \frac{5}{2}$

bump

$\log_e x = \frac{5}{2}$

$e^{\frac{5}{2}} = x$

$12.18 \approx x$

12. The pressure of a gas P, in atmospheres, varies inversely with the gas's volume V, in liters, and directly with the gas's temperature, T, in Kelvins. The gas has a pressure of 5 atmospheres if it has a volume of 20 liters and a temperature of 300 Kelvins.

a) Write a model for this variation.

12a. $P = \frac{\frac{1}{3}T}{V}$ or $\frac{T}{3V}$

$P = \frac{kT}{V} \to (5) = \frac{(300)k}{(20)} \to \frac{5}{15} = \frac{15k}{15} \to k = \frac{1}{3}$

15

1

b) Find the pressure of the gas if it has a temperature of 450 Kelvins and a volume of 60 liters.

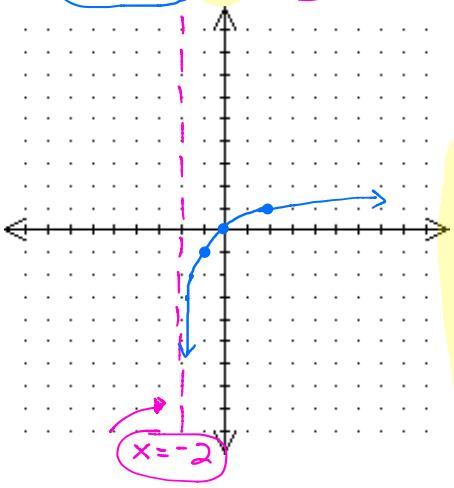
12b. $P = 2.5$ atmospheres of pressure

$P = \frac{T}{3V} = \frac{(450)}{3(60)} = \frac{450}{180} = 2.5$

Graph each function. State the domain, range, and asymptote(s). Show at least three points and the asymptote(s) in the graph.

$y = \log_2 x$ is $2^y = x$ (2 = bump)

13. $y = \log_2(x + 2) - 1$ — parent; Horizontal Shift Left 2; Vertical Shift Down 1



$2^y = x$

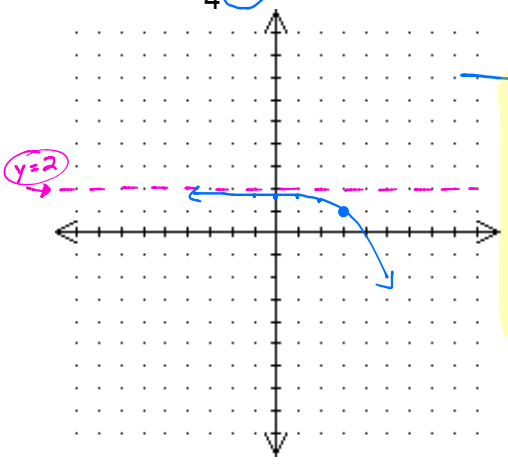
Left 2	X	Y	Down 1
$-1\frac{3}{4} = -2$	$\frac{1}{4}$	$-2 - 1 = -3$	
$-1\frac{1}{2} = -2$	$\frac{1}{2}$	$-1 - 1 = -2$	
$-1 = -2$	1	$0 - 1 = -1$	
$0 = -2$	2	$1 - 1 = 0$	
$2 = -2$	4	$2 - 1 = 1$	

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -2$

14. $y = -\frac{1}{4}(2)^{x-1} + 2$ — parent; Horizontal Shift Right 1; Vertical Shift Up 2



$y = 2^x$

Right 1	X	Y	Reflect and SHRINK	UP 2
$-1 = +1$	-2	$\frac{1}{4} \cdot -\frac{1}{4} = -\frac{1}{16} + 2 = 1\frac{15}{16}$ or 1.93		
$0 = +1$	-1	$\frac{1}{2} \cdot -\frac{1}{4} = -\frac{1}{8} + 2 = 1\frac{7}{8}$ or 1.88		
$1 = +1$	0	$1 \cdot -\frac{1}{4} = -\frac{1}{4} + 2 = 1\frac{3}{4}$ or 1.75		
$2 = +1$	1	$2 \cdot -\frac{1}{4} = -\frac{1}{2} + 2 = 1\frac{1}{2}$ or 1.5		
$3 = +1$	2	$4 \cdot -\frac{1}{4} = -1 + 2 = 1$		

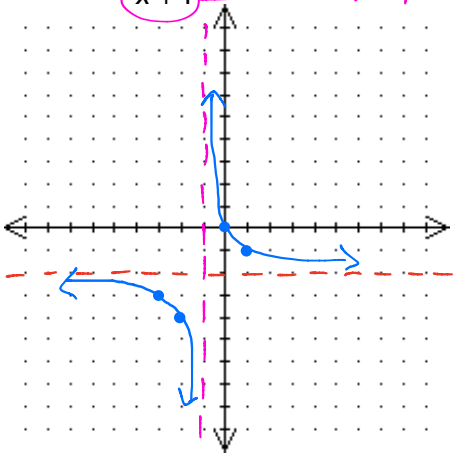
Domain: $(-\infty, \infty)$

Range: $(-\infty, 2)$

Asymptote: $y = 2$

Transformations:
1. Reflect over x-axis
2. Vertical SHRINK by $\frac{1}{4}$
3. Horiz. Shift RIGHT 1
4. Vertical Shift UP 2

15. $y = \frac{2}{x+1} - 2$ — $y = -2$; $x \neq -1$



X	Y
-3	-3
-2	-4
-1	Undefined
0	0
1	-1

Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, -2) \cup (-2, \infty)$

Horizontal asymptote: $y = -2$

Vertical asymptote: $x = -1$

For questions 16 – 17, solve the equations. Check each solution.

16. $\frac{3x}{(x+1)} + \frac{6}{2x} = \frac{7}{x}$ — solve... clear out the denominator

LCD: $2 \cdot x \cdot (x+1)$

$\frac{3x \cdot 2x(x+1)}{(x+1)} + \frac{6 \cdot 2x(x+1)}{2x} = \frac{7 \cdot 2x(x+1)}{x}$

$6x^2 + 6(x+1) = 14(x+1)$

$6x^2 + 6x + 6 = 14x + 14$

$-14x \quad -14 \quad\quad -14x \quad -14$

$6x^2 - 8x - 8 = 0$

$2(3x^2 - 4x - 4) = 0$

$2(3x+2)(x-2) = 0$

$3x+2=0 \quad x=-\frac{2}{3}$; $x-2=0 \quad x=2$

16. $x = -\frac{2}{3}, \; x = 2$

★ Check answers for extraneous solutions!

17. $\frac{x}{2x+7} = \frac{x-5}{x-1}$ PROPORTION !!!

$x(x-1) = (2x+7)(x-5)$

$x^2 - x = 2x^2 - 10x + 7x - 35$

$-x^2 + x \quad -x^2 + x$

$0 = x^2 - 2x - 35$

$0 = (x-7)(x+5)$

$x = 7 \quad x = -5$

17. $x = 7, \; x = -5$

★ Check answers for extraneous solutions!

18. Simplify $\frac{3x^2 + x - 2}{x^2 + 3x + 2} \div \frac{2x}{x+2}$. State any restrictions on the variable.

$\frac{(3x-2)(x+1)}{(x+2)(x+1)} \div \frac{2x}{(x+2)}$

$x \neq -2 \quad x = -1 \quad\quad x \neq -2$

$\frac{(3x-2)}{(x+2)} \cdot \frac{(x+2)}{2x}$

$x \neq 0$

$\frac{(3x-2)}{2x}$

18. $\frac{3x-2}{2x}$

Restrictions: $x \neq -2, -1, 0$

★ Check all denominators!

For questions 19 – 20, simplify completely.

19. $\dfrac{5x-1}{x^2+2x-8} - \dfrac{6}{(x+4)}$

Must have a common denominator!

$(x+4)(x-2)$

$\dfrac{5x-1}{(x+4)(x-2)} - \dfrac{6}{(x+4)} \cdot \dfrac{(x-2)}{(x-2)}$

$\dfrac{5x-1}{(x+4)(x-2)} + \dfrac{-6(x-2)}{(x+4)(x-2)}$

$\dfrac{5x-1-6x+12}{(x+4)(x-2)}$

$\dfrac{-x+11}{(x+4)(x-2)}$

or $\dfrac{-(x-11)}{(x+4)(x-2)}$

19. $\dfrac{-x+11}{(x+4)(x-2)}$

20. $\dfrac{\frac{2}{x} - \frac{4x}{1}}{\frac{4}{x^3}}$ $\times\frac{x}{x}$ $= \dfrac{\frac{2}{x} - \frac{4x^2}{x}}{\frac{4}{x^3}} = \dfrac{\frac{2-4x^2}{x}}{\frac{4}{x^3}}$

$\dfrac{2-4x^2}{x} \cdot \dfrac{x^3}{4}$

$\dfrac{x^2(2-4x^2)}{4} \to \dfrac{2x^2(1-2x^2)}{4} = \dfrac{x^2(1-2x^2)}{2}$

or $\dfrac{x^2-2x^4}{2}$

20. $\dfrac{x^2(1-2x^2)}{2}$