Advanced Algebra - Chapter 6 Review

Name KEY Period

1. List all possible rational zeros of the function $f(x) = \underset{q}{\boxed{4}}x^3 + 2x^2 + 16x \underset{p}{\boxed{+ 8}}$. Do **not** find the zeros.

1. $x = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{1}{4}$

$\dfrac{\text{Factors of } p}{\text{Factors of } q}: \dfrac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2, \pm 4}$

$\pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{4}{1}, \pm \frac{8}{1}, \pm \frac{1}{2}, \pm \overset{①}{\frac{2}{2}}, \pm \overset{②}{\frac{4}{2}}, \pm \overset{④}{\frac{8}{2}}, \pm \frac{1}{4}, \pm \overset{(\frac{1}{2})}{\frac{2}{4}}, \pm \overset{①}{\frac{4}{4}}, \pm \overset{②}{\frac{8}{4}}$

2. Solve the following equation, giving exact answers: $x^{\textcircled{3}} - x^2 - 9x + 9 = 0$. **No Calc.**

2. $x = 1, 3, -3$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -9 & 9 \\ & \downarrow & 1 & 0 & -9 \\ \hline & 1 & 0 & -9 & \boxed{0} \\ & x^2 & x & c & \text{Rem.} \end{array}$$

Quadratic →

$x^2 - 9 = 0$

$+9 \quad +9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

$x = \boxed{1}, \boxed{3}, \boxed{-3}$

3 total zeros

3. Solve the following equation, giving exact answers: $x^3 - 2x^2 - 9x = -18$. **No Calc.**

3. $x = 2, 3, -3$

Check: ±1, ±2, ±3, ±6, ±9, ±18

$\underset{q}{\boxed{1}}x^{\textcircled{3}} - 2x^2 - 9x \underset{p}{\boxed{+18}} = 0$ ($+18 \quad +18$)

$$\begin{array}{r|rrrr} \cancel{1} & 1 & -2 & -9 & 18 \\ & \downarrow & 1 & -1 & -10 \\ \hline & 1 & -1 & -10 & \boxed{8} \end{array}$$

$$\begin{array}{r|rrrr} \cancel{-1} & 1 & -2 & -9 & 18 \\ & \downarrow & -1 & 3 & 6 \\ \hline & 1 & -3 & -6 & \boxed{24} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -9 & 18 \\ & \downarrow & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & \boxed{0} \\ & x^2 & x & c & \text{R.} \end{array}$$

$x^2 - 9 = 0$

$+9 \quad +9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

3 Total Zeros:

$x = \boxed{2}, \boxed{3}, \boxed{-3}$

4. Solve the following equation, giving exact answers: $x^4 + x^2 = 2$. **No Calc.**

4. $x = \pm 1, \pm i\sqrt{2}$

$x^{\textcircled{4}} + x^2 - 2 = 0$

Factor $(x^2 + 2)(x^2 - 1) = 0$

Solve

$x^2 + 2 = 0 \qquad x^2 - 1 = 0$

$-2 \quad -2 \qquad +1 \quad +1$

$\sqrt{x^2} = \sqrt{-2} \qquad \sqrt{x^2} = \sqrt{1}$

$x = \pm i\sqrt{2} \qquad x = \pm 1$

4 Total zeros:

$x = \boxed{1}, \boxed{-1}, \boxed{i\sqrt{2}}, \boxed{-i\sqrt{2}}$

5. Solve the following equation, giving exact answers: $x^2 - 12x = 28$. **No Calc.**

5. $x = 14$ and -2

$-28 \quad -28$

$x^2 - 12x - 28 = 0$

$(x - 14)(x + 2) = 0$

$x - 14 = 0 \qquad x + 2 = 0$

$x = 14 \qquad x = -2$

6. Solve the following equation, giving exact answers: $(x-2)^2 + \cancel{64} = 72$. **No Calc.**

$-64 \quad -64$

$\sqrt{(x-2)^2} = \sqrt{8}$

$x - 2 = \pm\sqrt{8}$

$x \cancel{-2} = \pm 2\sqrt{2}$

$+2 \quad +2$

$\boxed{x = 2 \pm 2\sqrt{2}}$

$\sqrt{8}$

$\sqrt{4}\sqrt{2}$

$2\sqrt{2}$

6. $X = 2 \pm 2\sqrt{2}$

7. Find the zeros and multiplicity of zeros of the function: $f(x) = 2x^{⑤} - 12x^4 + 18x^3$. **(No calc)**

Factor GCF $2x^3(x^2 - 6x + 9) = 0$

$2x^3(x-3)(x-3) = 0$

$x = 0$ multiplicity of 3

$\underbrace{x = 3 \qquad x = 3}_{\text{Mult. of 2}}$

7. $X = 0$ multiplicity of 3

$X = 3$ multiplicity of 2

5 TOTAL Zeros: $\boxed{0}$, $\boxed{0}$, $\boxed{0}$, $\boxed{3}$, $\boxed{3}$

8. Solve the following equation, giving exact answers: $x^4 + x^3 + 2x^2 + 4x = \cancel{8}$. **YES Calc.**

$-8 \quad -8$

$x^{④} + x^3 + 2x^2 + 4x - 8 = 0$

-2	1	1	2	4	-8
	↓	-2	2	-8	8
1	1	-1	4	-4	$\boxed{0}$
	↓	1	0	4	
Quadratic →	1	0	4	$\boxed{0}$	
	x^2	x	c	R.	

$x^2 + 4 = 0$

$-4 \quad -4$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$

8. $x = -2, 1, 2i, -2i$

4 TOTAL ZEROS: $X = \boxed{-2}, \boxed{1}, \boxed{2i}, \boxed{-2i}$

9. Write the following polynomial in standard form. Also classify it by number of terms and degree. Polynomial: $(\cancel{x^2} + 2x + 3) + {}^{-}\cancel{(x^2} + 5)$ Add the opposite

$2x + 8$

9. Standard form: $2x + 8$

Name by degree: Linear

Name by number of terms: Binomial

10. Write the following polynomial in standard form. Also classify it by number of terms and degree. Polynomial: $(6x^3 + 3x^2 \cancel{- 5x} - 1) + {}^{-}(7x^3 \cancel{+ 5x} + 6)$

$-1x^3 + 3x^2 + 5$

10. Standard form: $-x^3 + 3x^2 + 5$

Name by degree: Cubic

Name by number of terms: Trinomial

11. Write the following polynomial in standard form. Also classify it by number of terms and degree. Polynomial: $(2x + 3)(4x^2 - 10)$ Distribute

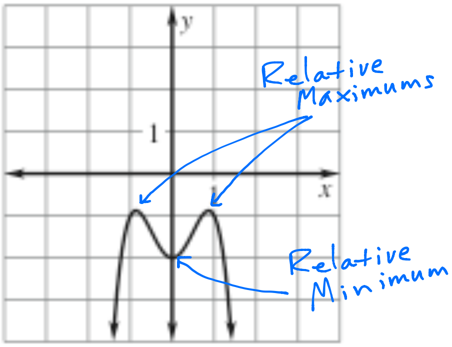
$8x^3 - 20x + 12x^2 - 30$

11. Standard form: $8x^3 + 12x^2 - 20x - 30$

Name by degree: Cubic

Name by number of terms: Polynomial

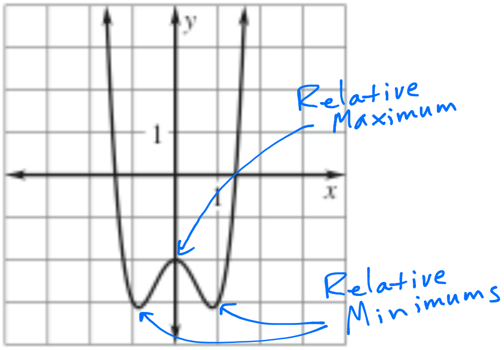
12. Use the graph below to **approximate** any relative minimums and maximums.



12. Max(s): $(-0.9, -0.9)$ and $(0.9, 0.9)$

Min(s): $(0, -2)$

13. Use the graph below to **approximate** any relative minimums and maximums.



13. Max(s): $(0, -2)$

Min(s): $(-0.9, -3.1)$ and $(0.9, -3.1)$

14. Divide $\underbrace{(x^4 + 9x^3 - 4x - 17)}_{\text{Look for a gap... missing an "}x^2\text{" term.}} \div (x + 5)$. **No Calc**

1x... synthetic

14. $x^3 + 4x^2 - 20x + 96 + \frac{-497}{x+5}$

$$\begin{array}{r} 3 \\ 96 \\ \times\ 5 \\ \hline 480 \end{array}$$

$$\begin{array}{r|rrrrr} -5 & 1 & 9 & 0 & -4 & -17 \\ & \downarrow & -5 & -20 & 100 & -480 \\ \hline & 1 & 4 & -20 & 96 & \boxed{-497} \\ & x^3 & x^2 & x & c & \text{Rem.} \end{array}$$

15. Divide $(12x^3 + 19x^2 + 8x + 6) \div (4x + 1)$. **No Calc**

15. $3x^2 + 4x + 1 + \frac{5}{4x+1}$

$3x^2 + 4x + 1 + \frac{5}{4x+1}$ ≠ 1x... Long divide

$$\begin{array}{r} 4x+1\ \big)\ \cancel{12x^3} + 19x^2 + 8x + 6 \\ +^-(\cancel{12x^3} + {}^-3x^2) \text{ drop} \\ \hline \cancel{16x^2} + 8x \\ +^-(\cancel{16x^2} + {}^-4x) \\ \hline \cancel{4x} + 6 \\ +^-(\cancel{4x} + {}^-1) \\ \hline \boxed{5} \end{array}$$

drop

Remainder

16. Three of the roots of a polynomial are -1, 5, –4*i*. What are all of the ***roots*** of this polynomial? Write the function in factored form. **No Calc.**

$(x-4i)(x+4i)$

$x^2+\cancel{4ix}-\cancel{4ix}-16i^2$

$+16$

16. Roots: $x = -1, 5, -4i, 4i$

Factored Form: $(x+1)(x-5)(x^2+16)$

17. Two of the roots of a polynomial are $-\sqrt{3}$ and 7*i*. What are all of the ***factors*** of this polynomial? Explain. **No Calc.**

$(x+\sqrt{3})(x-\sqrt{3})(x-7i)(x+7i)$

$x^2+\cancel{\sqrt{3}x}-\cancel{\sqrt{3}x}-\sqrt{9}$

x^2-3

$x^2+\cancel{7ix}-\cancel{7ix}-49i^2$

$x^2-49(-1)$

x^2+49

17. Factors: $(x^2-3)(x^2+49)$

Explanation:

Irrational Root Theorem

Complex Root Theorem . . .

i's and sq. roots come in pairs.

ODD positive

ODD negative

Negative

ODD degree

18. Describe the end behavior of the function

$f(x) = -2x^{5} - 8x^4 + 10x^3$ by filling in the blanks at right. **No Calc.**

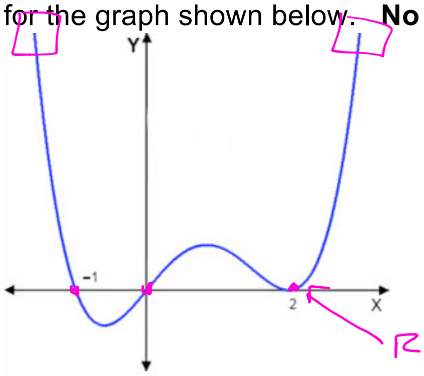
18. $\lim_{x\to-\infty} f(x) = \infty$ UP

$\lim_{x\to+\infty} f(x) = -\infty$ DOWN

19. Write a possible function in factored form for the graph shown below. **No Calc.**

19. $f(x) = x(x+1)(x-2)^2$

$x = -1, 0, 2$ (multiplicity 2)



Repeated Zero Touches x-axis and bounces back

20. Describe the end behavior of the graph in #19. **No Calc**

20. $\lim_{x\to-\infty} f(x) = \infty$ UP

$\lim_{x\to+\infty} f(x) = \infty$ UP

21. Determine if $(x - 4)$ is a factor of the function $f(x) = x^4 - 3x^2 + 5x - 8$. How does this method shown if this or is not a factor? **No Calc.**

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & -3 & 5 & -8 \\ & \downarrow & 4 & 16 & 52 & 228 \\ \hline & 1 & 4 & 13 & 57 & \boxed{220} \end{array}$$

21. 4 is not a zero and $(x-4)$ is not a factor.

The Remainder Theorem says the remainder must be zero after dividing by the possible factor.

22. The average amount of tangarines (t in pounds) eaten per person each year in the United States from 2001 to 2006 can be modeled by $t = 0.298y^3 - 1.73y^2 + 2.05y + 4.45$ where y is the number of years since 2001. **Using your graphing calculator:**

a. Graph the function and identify the relative minimum and relative maximum where $0 \le y \le 4$.

Relative minimum: $(3.14, 3.06)$

Relative maximum: $(0.73, 5.14)$

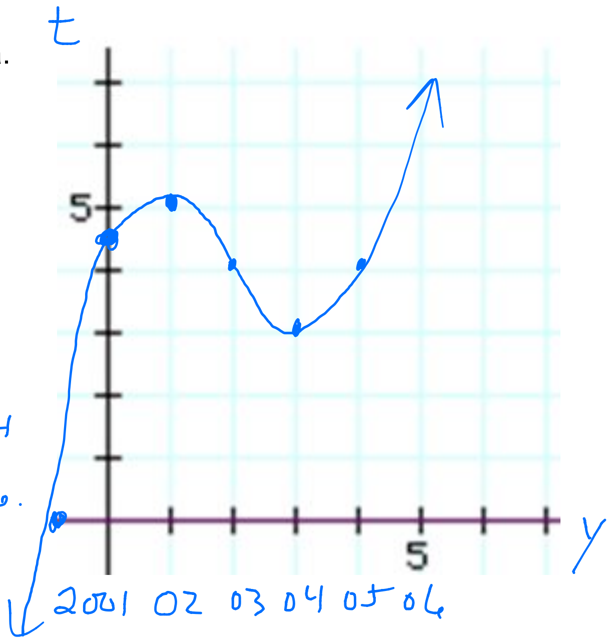
b. What is the real-life meaning of the relative minimum?

2004 was the smallest avg. amount of tangerines eaten from 2001-2006.

c. What is the real-life meaning of the relative maximum?

2002 was the largest avg. amount of tangerines eaten from 2001-2006.

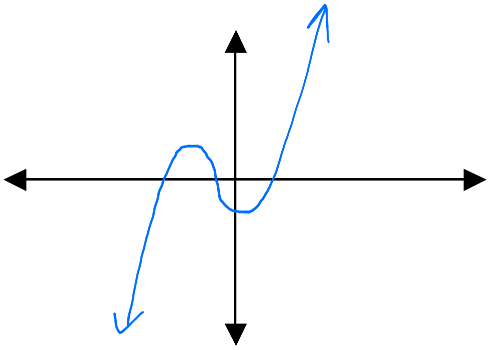
a.



23. **Use your graphing calculator** to sketch a graph on the interval -5 ≤ x ≤ 3 and find the coordinates of the zero(s), relative maximum(s), and relative minimums(s) of the function listed below. Also identify the end behavior of the graph of the function.

Function: $f(x) = 0.25x^3 + 0.755x^2 - 1.06x - 1.17$

a. Sketch:



b. Zero(s) of the function: $x = -3.81, -.78, 1.57$

c. Relative minimum(s): (0.55, -1.48)

d. Relative maximum(s): (-2.56, 2.30)

e. End behavior:

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = \infty$