Cumulative Review Name KEY

1. a) Convert to radians: 100°

$\frac{\cancel{100}^{5}}{1}\cdot\frac{\pi}{\cancel{180}_{9}} = \frac{5\pi}{9}$ RAD.

b) Convert to degrees: $\frac{7\pi}{15}$

$\frac{7\cancel{\pi}}{\cancel{15}_{1}}\cdot\frac{\cancel{180}^{12}}{\cancel{\pi}} = 84°$

2. The angle subtended by an arc is 54° and the radius is 8 feet,

a) find the arc length.

$\frac{\cancel{54}^{27}}{1}\cdot\frac{\pi}{\cancel{180}_{90}} = \frac{27\pi}{90} = \frac{3\pi}{10}$

$s = \theta r$ (radians)

$s = \left(\frac{3\pi}{10}\right)(8) = 7.54$ ft.

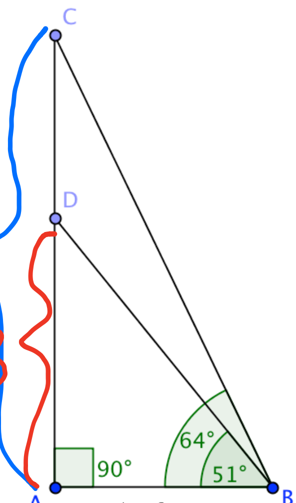
b) find the area of the sector.

$A = \frac{1}{2}\theta r^2$

$\left(\frac{1}{2}\right)\left(\frac{3\pi}{10}\right)\left(\frac{64}{1}\right)$

30.16 ft.^{2}

3. Use the picture below:



150 ft.

a) Find the distance from point A to point C.

$\frac{\tan 64°}{1} = \frac{x}{150}$

307.55 ft.

b) Find the distance between point C and point D.

$\frac{\tan 51°}{1} = \frac{x}{150}$

$x = 185.23$

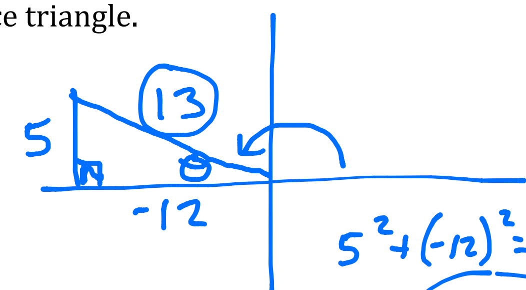
307.55
− 185.23
122.32 ft.

307.55

185.23

4. Let θ be in standard position with the point (-12, 5) on it's terminal side.

a) Draw the terminal side of the angle and the reference triangle.



$5^2 + (-12)^2 = x^2$

$13 = x$

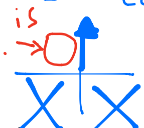
b) Find the values of the six trig functions.

$\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{-12}{13}$ $\tan\theta = \frac{-5}{12}$

$\csc\theta = \frac{13}{5}$ $\sec\theta = \frac{-13}{12}$ $\cot\theta = \frac{-12}{5}$

5. $\sin\theta = \frac{\sqrt{5}}{4}$ O/H, $\cot\theta < 0$

cot is neg.



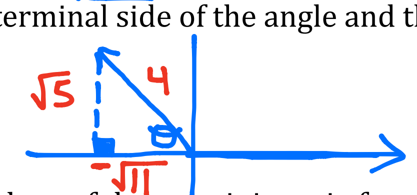
Sine is positive in I, II

cot is negative in II, IV

a) Name the quadrant that θ is in.

QII

b) Draw the terminal side of the angle and the reference triangle.



$x^2 + (\sqrt{5})^2 = (4)^2$

$x^2 + 5 = 16$

$x^2 = 11$

$x = \sqrt{11}$

c) Find the values of the remaining trig functions.

$\cos\theta = \frac{-\sqrt{11}}{4}$ $\tan\theta = \frac{\sqrt{5}}{-\sqrt{11}} = \frac{-\sqrt{55}}{11}$

$\csc\theta = \frac{4\sqrt{5}}{5}$ $\sec\theta = \frac{4}{-\sqrt{11}} = \frac{-4\sqrt{11}}{11}$ $\cot\theta = \frac{-\sqrt{11}}{\sqrt{5}} = \frac{-\sqrt{55}}{5}$

6. Evaluate the following. Give exact answers.

a) $\tan 210°$

$\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

b) $\cos\left(-\frac{4\pi}{3}\right) + 2\pi$

OR $\cos\left(\frac{2\pi}{3}\right)$

$-\frac{1}{2}$

c) $\csc^2\frac{2\pi}{3}$

$\left(\csc\frac{2\pi}{3}\right)^2$

$= \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$

d) $\cot\frac{7\pi}{2} + \sec\frac{\pi}{3}$

$\frac{3\pi}{2}$

$\frac{1}{\cos\frac{\pi}{3}}$

$\frac{0}{-1} + \frac{1}{\frac{1}{2}}$

$0 + 2 = 2$

7. $\dfrac{2\tan\frac{\pi}{2}}{1-\tan^2\frac{\pi}{2}}$

a) Write the expression as sine, cosine, or tangent of a single angle. $\tan\left(2\cdot\frac{\pi}{2}\right)$ $\tan(\pi)$

b) Find the exact value of the expression.

$\tan\pi = \boxed{0}$

8. Find the exact value of $\cos(165°) = \cos(\underset{\alpha}{135°} + \underset{\beta}{30°}) = \cos 135°\cos 30° - \sin 135°\sin 30°$

$-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

9. Given $\sin\alpha = \frac{1}{2}$ (O/H), $0 < \alpha < \frac{\pi}{2}$ and $\cos\beta = \frac{3}{4}$, $0 < \beta < \frac{\pi}{2}$

a) find $\tan(\alpha - \beta)$

$\frac{\sqrt{3}-\sqrt{7}}{3+\sqrt{21}}$

See below

b) find $\sin(2\alpha)$

$\frac{\sqrt{3}}{2}$

c) find $\cos\frac{\beta}{2}$

$+\frac{\sqrt{14}}{4}$ or $+\sqrt{\frac{7}{8}}$

10. Verify each identity:

a) $\sin\theta\tan\theta + \cos\theta = \sec\theta$

$\frac{\sin\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1}$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1}\cdot\frac{\cos\theta}{\cos\theta}$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$ Pythagorean Identity

$\frac{1}{\cos\theta}$

$\sec\theta = \sec\theta$ //

b) $\dfrac{\sin(\alpha+\beta)}{\tan\alpha + \tan\beta} = \cos\alpha\cos\beta$ Formula

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{(\tan\alpha + \tan\beta)}$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\cos\beta}{\cos\beta}\cdot\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}\cdot\frac{\cos\alpha}{\cos\alpha}}$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}} \cdot \frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta + \cancel{\cos\alpha\cos\beta}}$

$\cos\alpha\cos\beta = \cos\alpha\cos\beta$ //

c) $\frac{(1+\cos\theta)}{(1+\cos\theta)}\frac{1}{1} - \dfrac{\sin^2\theta}{1+\cos\theta} = \cos\theta$

$\frac{1+\cos\theta}{1+\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta}$

$\frac{1+\cos\theta - \sin^2\theta}{1+\cos\theta}$

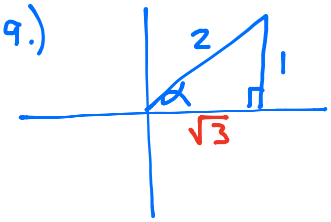
$\frac{1-\sin^2\theta + \cos\theta}{1+\cos\theta}$ Identity

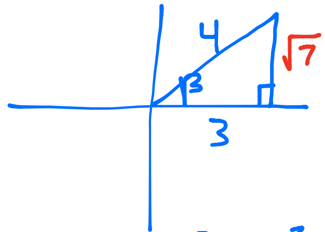
$\frac{\cos^2\theta + \cos\theta}{1+\cos\theta}$

Factor $\frac{\cos\theta(\cancel{\cos\theta+1})}{\cancel{1+\cos\theta}}$

$\cos\theta = \cos\theta$ //

9.)





$$1^2 + x^2 = 2^2$$
$$1 + x^2 = 4$$
$$x^2 = 3$$
$$x = \sqrt{3}$$

$$\boxed{\begin{aligned} \sin\alpha &= \tfrac{1}{2} \\ \cos\alpha &= \tfrac{\sqrt{3}}{2} \\ \tan\alpha &= \tfrac{1}{\sqrt{3}} \text{ or } \tfrac{\sqrt{3}}{3} \end{aligned}}$$

$$3^2 + x^2 = 4^2$$
$$9 + x^2 = 16$$
$$x^2 = 7$$
$$x = \sqrt{7}$$

$$\boxed{\begin{aligned} \sin\beta &= \tfrac{\sqrt{7}}{4} \\ \cos\beta &= \tfrac{3}{4} \\ \tan\beta &= \tfrac{\sqrt{7}}{3} \end{aligned}}$$

9a.) $\tan(\alpha - \beta)$

$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\frac{\frac{\sqrt{3}}{3} - \frac{\sqrt{7}}{3}}{1 + \left(\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{7}}{3}\right)}$$

$$\frac{\frac{\sqrt{3}-\sqrt{7}}{3}}{\frac{3}{3}1 + \frac{\sqrt{21}}{3}}$$

$$\frac{\frac{\sqrt{3}-\sqrt{7}}{\cancel{3}}}{\frac{3+\sqrt{21}}{\cancel{3}}}$$

$$\boxed{\frac{\sqrt{3}-\sqrt{7}}{3+\sqrt{21}}}$$

OK to stop here.

9b.) $\sin(2\alpha)$

$$2\sin\alpha\cos\alpha$$

$$\frac{2}{1} \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$$

$$\frac{{}^{1}\cancel{2}\sqrt{3}}{{}_{2}\cancel{4}}$$

$$\boxed{\frac{\sqrt{3}}{2}}$$

9c.) $\cos\left(\frac{\beta}{2}\right)$

Half angle is in QI → positive cosine

$$\pm\sqrt{\frac{1+\cos\beta}{2}}$$

$$+\sqrt{\frac{1+\frac{3}{4}}{2}}$$

$$+\sqrt{\frac{\frac{4}{4}+\frac{3}{4}}{2}}$$

$$+\sqrt{\frac{\frac{7}{4}}{2}}$$

$$+\sqrt{\frac{7}{4}\cdot\frac{1}{2}}$$

$$+\sqrt{\frac{7}{8}} = +\frac{\sqrt{7}}{\sqrt{8}} = \frac{\sqrt{7}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \boxed{\frac{\sqrt{14}}{4}}$$