Term 1 Final Review

Name KEY - Block 2

Convert the angle in radians to degrees.

1) $\frac{11\pi}{12} \cdot \frac{180}{\pi} = 11 \cdot 15$

1) $165°$

Convert the angle in degrees to radians. Express the answer as multiple of π.

2) $-54° \cdot \frac{\pi}{180} = \frac{6\pi}{20} = -\frac{3\pi}{10}$

2) $-\frac{3\pi}{10}$

$s = \theta r$

$\theta \Rightarrow$ RADIANS

If s denotes the length of the arc of a circle of radius r subtended by a central angle θ, find the missing quantity.

3) r = 13.9 inches, θ = 150°, s = ?

$150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$ $s = r \cdot \theta = 13.9 \cdot \frac{5\pi}{6} = 36.4$ in

3) 36.4 in

4) $r = \frac{1}{4}$ feet, s = 6 feet, θ = ?

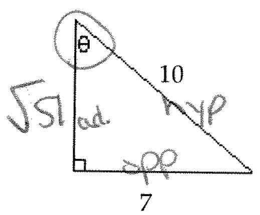
$s = \theta r$

$4 \cdot 6 = \theta \cdot \frac{1}{4} \cdot 4$ $\theta = 24$ rad.

4) 24 rad.

Find the value of the indicated trigonometric function of the angle θ in the figure. Give an exact answer with a rational denominator.

5)



Find cos θ.

$10^2 = 7^2 + y^2$

$100 = 49 + y^2$

$51 = x^2$

$\sqrt{51}$

$\cos\theta = \frac{\sqrt{51}}{10}$

5) $\frac{\sqrt{51}}{10}$

Use identities to find the exact value of the indicated trigonometric function of the acute angle θ.

6) $\sin\theta = \frac{1}{4}$, $\cos\theta = \frac{\sqrt{15}}{4}$ Find cot θ. $= \frac{\cos\theta}{\sin\theta} = \frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = \sqrt{15}$

6) $\sqrt{15}$

Use Fundamental Identities and/or the Complementary Angle Theorem to find the exact value of the expression. Do not use a calculator.

7) $\sec^2 80° - \tan^2 80°$ $\frac{1}{\cos^2 80} - \frac{\sin^2 80}{\cos^2 80} = \frac{1 - \sin^2 80}{\cos^2 80} = \frac{\cancel{\sin^2 80} + \cos^2 80 - \cancel{\sin^2 80}}{\cos^2 80}$

$\sec^2 = \cancel{1 + \tan^2} - \cancel{\tan^2}$ 1

7) 1

8) cos 40° sec 40°

$\cancel{\cos 40} \cdot \frac{1}{\cancel{\cos 40}} = 1$

8) 1

Use the definition or identities to find the exact value of the indicated trigonometric function of the acute angle θ.

9) $\tan\theta = \frac{4}{3}$ $\frac{opp}{adj}$ $\frac{y}{x}$ Find cos θ.

5 hyp, 4, θ adj, 3

$\frac{3}{5}$

9) 3/5

Find the exact value. Do not use a calculator.

10) $\cot\frac{\pi}{4}$ $\frac{x}{y} = 1$

UNIT CIRCLE

10) 1

11) sec 60°

$\hookrightarrow \cos = \frac{1}{2}$

X

2

11) 2

* $A = \frac{1}{2} r^2 \theta$

If A denotes the area of the sector of a circle of radius r formed by the central angle θ, find the missing quantity. If necessary, round the answer to two decimal places.

12) $r = 4$ feet, $A = 47$ square feet, $\theta = ?$

$47 = \frac{1}{2} \cdot 16 \cdot \theta$ $\quad \frac{47}{8} = \frac{8\theta}{8}$ $\quad \theta = 5.88$ rad

12) 5.88 rad.

Solve the problem.

13) A building 230 feet tall casts a 100 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

$\tan x = \frac{230}{100}$ $\quad \tan^{-1}\left(\frac{230}{100}\right) = x$

100

13) $66.50°$

A point on the terminal side of angle θ is given. Find the exact value of the indicated trigonometric function.

14) (3, 4) Find csc θ.

5, 4, 3, θ

↳ sin $\frac{5}{4}$

14) $5/4$

Find the exact value of the indicated trigonometric function of θ.

15) $\cos\theta = \frac{21}{29}$, $\frac{3\pi}{2} < \theta < 2\pi$ Find cot θ. $= \frac{adj}{opp}$ $\quad \frac{21}{-20}$

15) $\frac{-21}{20}$

Solve the problem.

16) For what numbers x, $0 \le x \le 2\pi$, does $\cos x = 0$?

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

16) $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Use transformations to graph the function.

17) $y = -2 \cos \frac{1}{3}x$

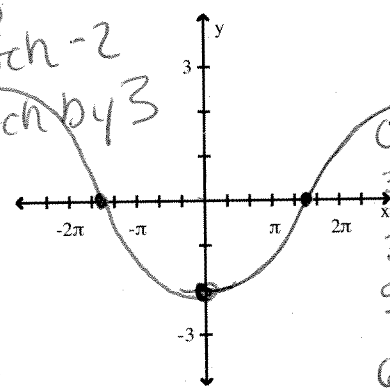
$a = 2$ $\quad p = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

17)

flip

vert. stretch -2

hort. stretch by 3



x		y
0	$3 \cdot 0$	$1 \cdot -2 = -2$
$\frac{3\pi}{2}$	$3 \cdot \frac{\pi}{2}$	$0 \cdot -2 = 0$
3π	$3 \cdot \pi$	$-1 \cdot -2 = 2$
$\frac{9\pi}{2}$	$3 \cdot \frac{3\pi}{2}$	$0 \cdot -2 = 0$
6π	$3 \cdot 2\pi$	$1 \cdot -2 = -2$

Without graphing the function, determine its amplitude or period as requested.

18) $y = -3 \sin \frac{1}{4}x$ Find the amplitude.

$a = 3$

18) $a = 3$

19) $y = \cos 3x$ Find the period.

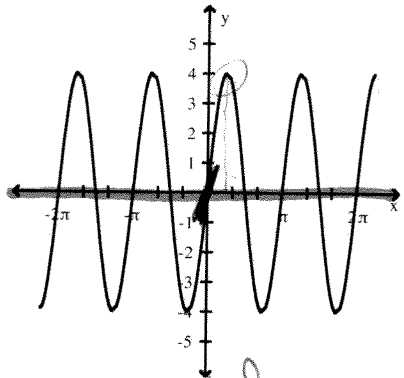
$p = \frac{2\pi}{3}$

19) $p = \frac{2\pi}{3}$

$P = \frac{2\pi}{\omega}$ $\pi = \frac{2\pi}{\omega}$ $\omega = 2$ $\frac{\pi \cdot \omega}{\pi} = \frac{2\pi}{\pi}$ $\omega = 2$

Find an equation for the graph.

20)



$a = 4$
$\omega = 2$
$h =$
$x = 0$

$y = 4\sin(2x)$

$y = 4\cos\left(2\left(x - \frac{\pi}{3}\right)\right)$

20) ____________

→ horizontal

Find the phase shift.

21) $y = -3\sin\left(x - \frac{\pi}{4}\right)$ Right $\frac{\pi}{4}$

21) ____________

Find the exact value of the expression.

22) $\tan^{-1}(-\sqrt{3})$ $\frac{y}{x} = \frac{-\sqrt{3}}{1}$ $\frac{-\pi}{3}$ $\cancel{\frac{5\pi}{3}}$

22) ____________

answer will be in terms of π

23) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $y=$ $\frac{\pi}{4}$

23) ____________

Find the exact value of the expression. Do not use a calculator.

24) $\cos^{-1}\left(\cos\frac{\pi}{2}\right) = \cos^{-1}(0)$ $x=$ where $x=0$? $\frac{\pi}{2}$ $\cancel{\frac{3\pi}{2}}$

24) ____________

Find the exact value of the expression.

25) $\sin\left[\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right] = \sin\left(\frac{3\pi}{4}\right)$ where $x=$ $\frac{\sqrt{2}}{2}$

25) ____________

Simplify the expression.

26) $\frac{\cos\theta}{1+\sin\theta} + \frac{\tan\theta}{1} \cdot \frac{(1+\sin\theta)}{1+\sin\theta}$

26) ____________

$$\frac{\cos\theta + \tan\theta(1+\sin\theta)}{1+\sin\theta}$$

$$\frac{\cos\theta + \frac{\sin\theta}{\cos\theta}(1+\sin\theta)}{1+\sin\theta}$$

$$\frac{\frac{\cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} + \frac{\sin\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}}{1+\sin\theta}$$

$$\frac{\frac{\boxed{\cos^2\theta} + \sin\theta + \boxed{\sin^2\theta}}{\cos\theta}}{1+\sin\theta}$$

$$\frac{\cancel{1+\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{1+\sin\theta}} = \frac{1}{\cos\theta} = \sec\theta$$

Establish the identity.

27) $(1 + \tan 2u)(1 - \sin 2u) = 1$

$\sec^2\theta(1 - \sin^2\theta) = 1$

$\sec^2\theta(\cos^2\theta + \cancel{\sin^2\theta} - \cancel{\sin^2\theta}) = 1$

$\frac{1}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta}$

1

27) 1

28) $\tan\theta \cdot \csc\theta = \sec\theta$

$\frac{\cancel{\sin\theta}}{\cos\theta} \cdot \frac{1}{\cancel{\sin\theta}} = \frac{1}{\cos\theta} = \sec\theta$

28) $\sec\theta$

Find the exact value of the expression.

29) $\sin 165° = \sin(135 + 30)$

$= \sin 135 \cdot \cos 30 + \cos 135 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + -\frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

29) $\frac{\sqrt{6} - \sqrt{2}}{4}$

30) $\cos 15° \cos 45° - \sin 15° \sin 45°$

$\alpha \quad \beta$

$= \cos(\alpha + \beta) = \cos(15 + 45)$

$\cos(60) \quad \frac{1}{2}$

30) $\frac{1}{2}$

Solve the equation on the interval $0 \le \theta < 2\pi$.

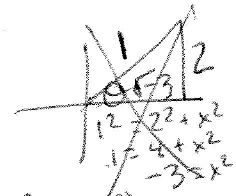
31) $2\cos\theta + 3 = 2$

$-3 \quad -3$

$\frac{2\cos\theta}{2} = \frac{-1}{2}$

$\cos\theta = -\frac{1}{2}$

$\theta = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$



31) $\frac{2\pi}{3}, \frac{4\pi}{3}$

32) $2\sin^2\theta - 3\sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0 \qquad \sin\theta - 2 = 0$

$\sin\theta = -\frac{1}{2} \qquad \sin\theta = 2 \quad y = \frac{2}{1}$ undefinded

$\frac{y}{2}$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

32) $\frac{7\pi}{6}, \frac{11\pi}{6}$

Solve the problem.

33) A twenty-five foot ladder just reaches the top of a house and forms an angle of 41.5° with the wall of the house. How tall is the house? Round your answer to the nearest 0.1 foot.

$25 \cos 41.5° = \frac{x}{25} \cdot 25$

$18.7 = x$

33) 18.7 ft.

Solve the triangle.

34) $B = 40°$, $C = 20°$, $a = 2$

34) other sheet

Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

35) $a = 7$, $b = 5$, $B = 20°$

35) other sheet

Solve the triangle.

36) $a = 7$, $b = 13$, $c = 15$

36) other sheet

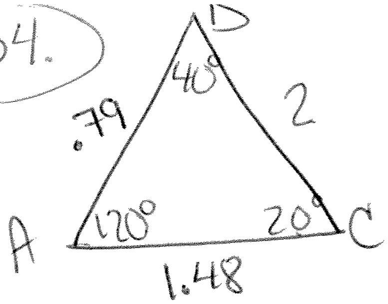
Find the area of the triangle. If necessary, round the answer to two decimal places.

37) $A = 83°$, $b = 9$, $c = 6$

37) other sheet

34.

1 △



$$\frac{\sin 120}{2} = \frac{\sin 40}{b}$$

$$\frac{b\sin 120}{\sin 120} = \frac{2\sin 40}{\sin 120}$$

$$b = 1.48$$

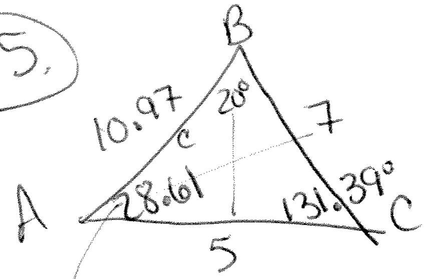
$$\frac{\sin 120}{2} = \frac{\sin 20}{c}$$

$$\frac{c\sin 120}{\sin 120} = \frac{2\sin 20}{\sin 120}$$

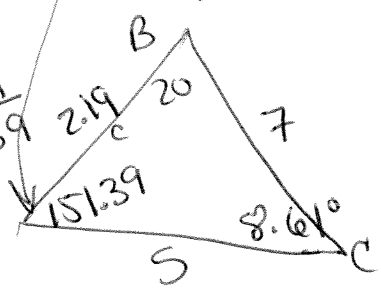
$$c = .79$$

35.

2 △s



$$\begin{array}{r} 180 \\ -28.61 \\ \hline 151.39 \end{array}$$



$$\frac{\sin 20}{5} = \frac{\sin A}{7}$$

$$\frac{5\sin A}{5} = \frac{7\sin 20}{5}$$

$$\cancel{\sin} A = \sin^{-1}\left(\frac{7\sin 20}{5}\right)$$

$$A = 28.61°$$

$$\frac{\sin 20}{5} = \frac{\sin 131.39}{c}$$

$$\frac{c\sin 20}{\sin 20} = \frac{5\sin 131.39}{\sin 20}$$

$$c = 10.97$$

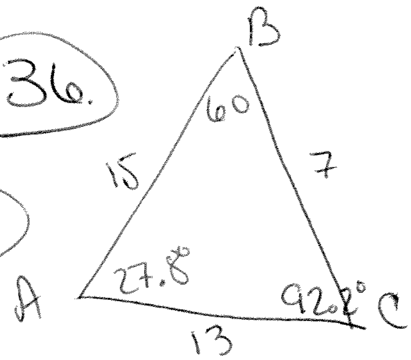
$$\frac{\sin 20}{5} = \frac{\sin 8.61}{c}$$

$$\frac{c\sin 20}{\sin 20} = \frac{5\sin 8.61}{\sin 20}$$

$$c = 2.19$$

36.

1 △



$$7^2 = 15^2 + 13^2 - 2(15)(13)\cos(A)$$

$$49 = 225 + 169 - 390\cos A$$

$$49 = 394 - 390\cos A$$

$$-394 \quad -394$$

$$\frac{-345}{-390} = \frac{-390}{-390}\cos A$$

$$\cos^{-1}\left(\frac{-345}{-390}\right) = A$$

$$27.8° = A$$

$$13^2 = 15^2 + 7^2 - 2(15)(7)\cos B$$

$$169 = 225 + 49 - 210\cos B$$

$$169 = 274 - 210\cos B$$

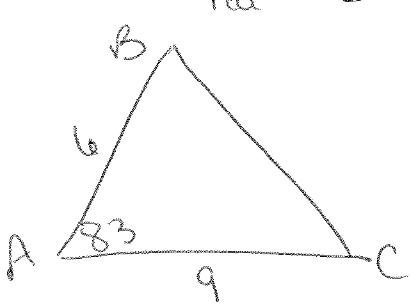
$$-105 = -210\cos B$$

$$.5 = \cos B$$

$$\cos^{-1}(.5) = B$$

37.

$$A_{rea} = \frac{1}{2} 6 \cdot 9 \cdot \sin 83 = 27\sin 83$$



Area = 26.8 units²