Pre-Calculus – College Algebra Review for Semester 2 Final Name ______________________________

The following is a list of topics to know for your final. Below that are problems to complete that will you review. The chapter(s) the problem is from is given in parentheses. It is also a good idea to look at old homework, notes, videos, and quizzes to study.

Chapter 3 Topics
Determine what is a function
Determine what is even, odd, or neither
Evaluate and graph piecewise functions
Find the domain of a function
Perform function operations
Find local maximums and minimums
Find domain and range from a graph
Find the increasing and decreasing intervals of a graph
Evaluate functions
Graph base graphs and transformations

Chapter 4 Topics
Find the vertex of a quadratic equation
Find the max or min of a quadratic equation
Know vertex and standard form of a quadratic
Go between vertex and standard form.
Solve quadratic inequalities
Solve application problems (revenue, bridges, etc.)
Graph from standard and vertex form

Chapter 5 Topics
Know factored and standard form of a polynomial
Graph a polynomial
Find increasing and decreasing intervals of a graph
Find zeros and multiplicity of a polynomial
Determine whether a graph crosses or touches at zeros
List all possible rational zeros
Prove zeros and factor polynomials with synthetic division
Write a polynomial in standard form given the zeros
Find a remainder
Find zeros, local max and mins from your calculator
Work with complex (imaginary) zeros
Find the degree and maximum number of turning points
Solve polynomial inequalities
Graph rational equations
Find vertical and end behavior asymptotes
Find the limits of a rational graph
Find the domain and range of a rational equation
Solve rational inequalities

Chapter 6 Topics
Find and evaluate a composite function
Find an inverse function
Find the domain and range of a function and inverse
Graph an inverse function
Prove two functions are inverses
Find an exponential equation given two points
Solve exponential equations
Solve logarithmic equations
Convert exponential and logarithmic expressions
Evaluate logarithms
Expand and condense logarithms
Use the compound interest formula
Use the continuous interest formula

Chapter 11 Topics
Know the equations of conic sections
Graph conic sections given the equation
Graph conic sections given facts
Find the equation of a conic section given graph
Find the equation of a conic section given facts
Change from general form to standard form
Determine the conic given general form
Find the focus of a paraboloid

1. (3) Find $f(x+1)$ given $f(x) = 3x^2 + 1$

$$f(x+1) = 3(x+1)^2 + 1$$

$$= 3(x^2 + 2x + 1) + 1$$

$$= 3x^2 + 6x + 3 + 1$$

$$= 3x^2 + 6x + 4$$

2. (3) Determine $\frac{g}{f}(x)$ given $f(x) = x + 2$ and $g(x) = \sqrt[3]{x}$

$$\frac{g}{f}(x) = \frac{\sqrt[3]{x}}{x+2}$$

3. (3), (5) Find the domain and range of the function $f(x) = \frac{5x+2}{x^2+5x-36}$

To find the domain, we look at the vertical asymptote(s) (where denominator equals zero)

$$x^2 + 5x - 36 \neq 0$$

$$(x+9)(x-4) \neq 0$$

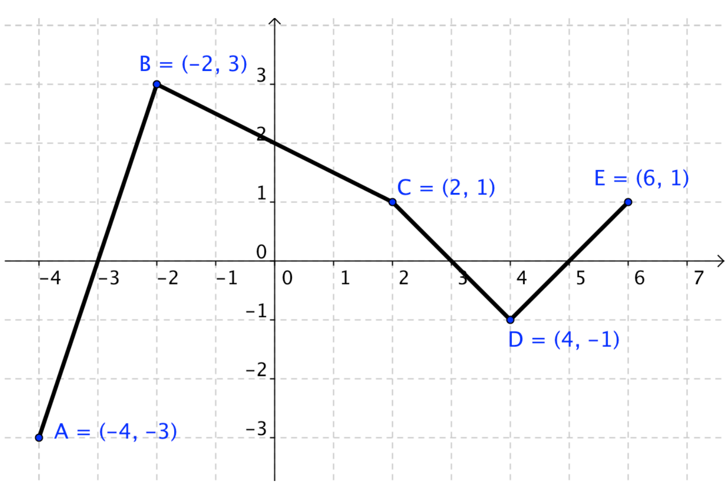
$$\Rightarrow \{x | x \neq -9, x \neq 4\}$$

To find the range, we look at the horizontal asymptote

$$\lim_{x \to \infty} \frac{5x+2}{x^2+5x-36} \approx \frac{5\infty + 2}{\infty^2 + 5x - 36} \approx \frac{1}{\infty} = 0$$

$$\Rightarrow \{y | y \neq 0\}$$

4. (3) Use the graph to determine:



a) Local Maximum(s)   b) Local Minimum(s)

c) Increasing Interval   d) Decreasing Interval

e) Whether the function is even, odd, or neither

f) Where f(x) ≥ 0 g) Where f(x) < 0

a) Max of 3 at x = -2 b) Min of -1 at x = 4

c) Inc: (-4, -2) U (4, 6) d) Dec: (-2, 4)

e) Neither because there is no symmetry

f) [-3, 3] U [5, 6] g) (-4, -3) U (3, 5)

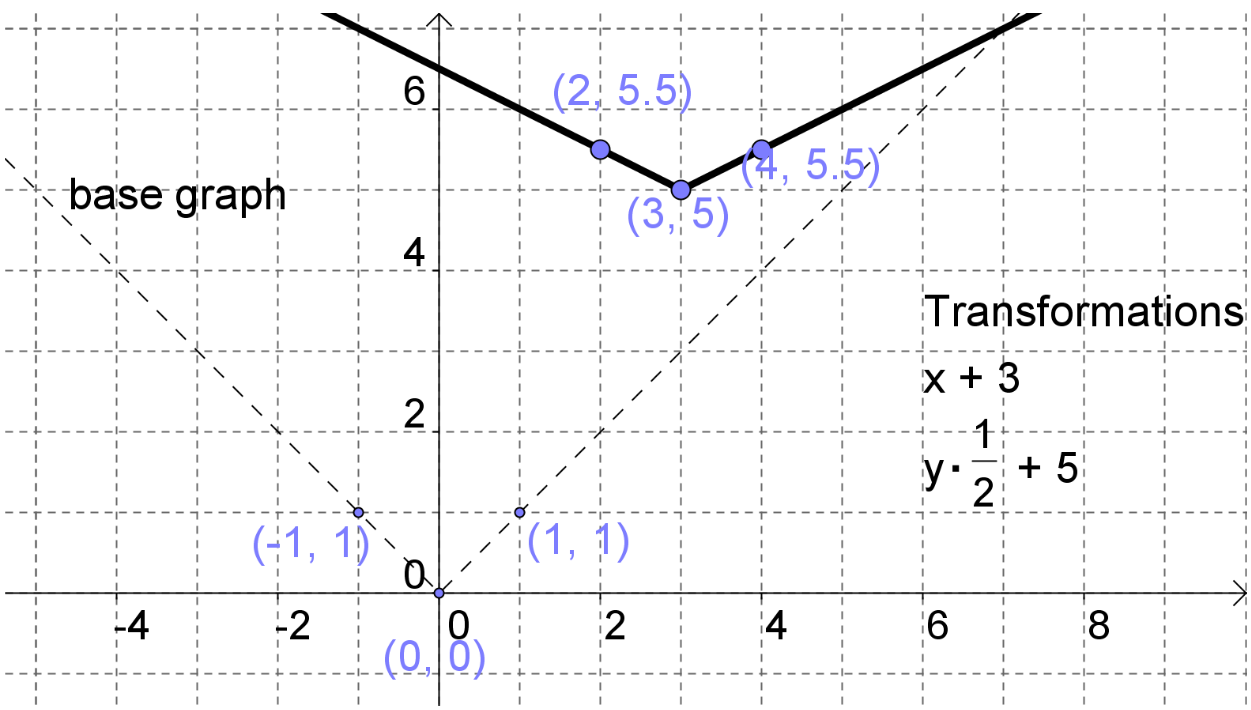
5. (3) Find the domain of the function $f(x) = \frac{x^3-1}{\sqrt{x+2}}$

The denominator cannot equal zero and we cannot do the square root of a negative number, so we solve

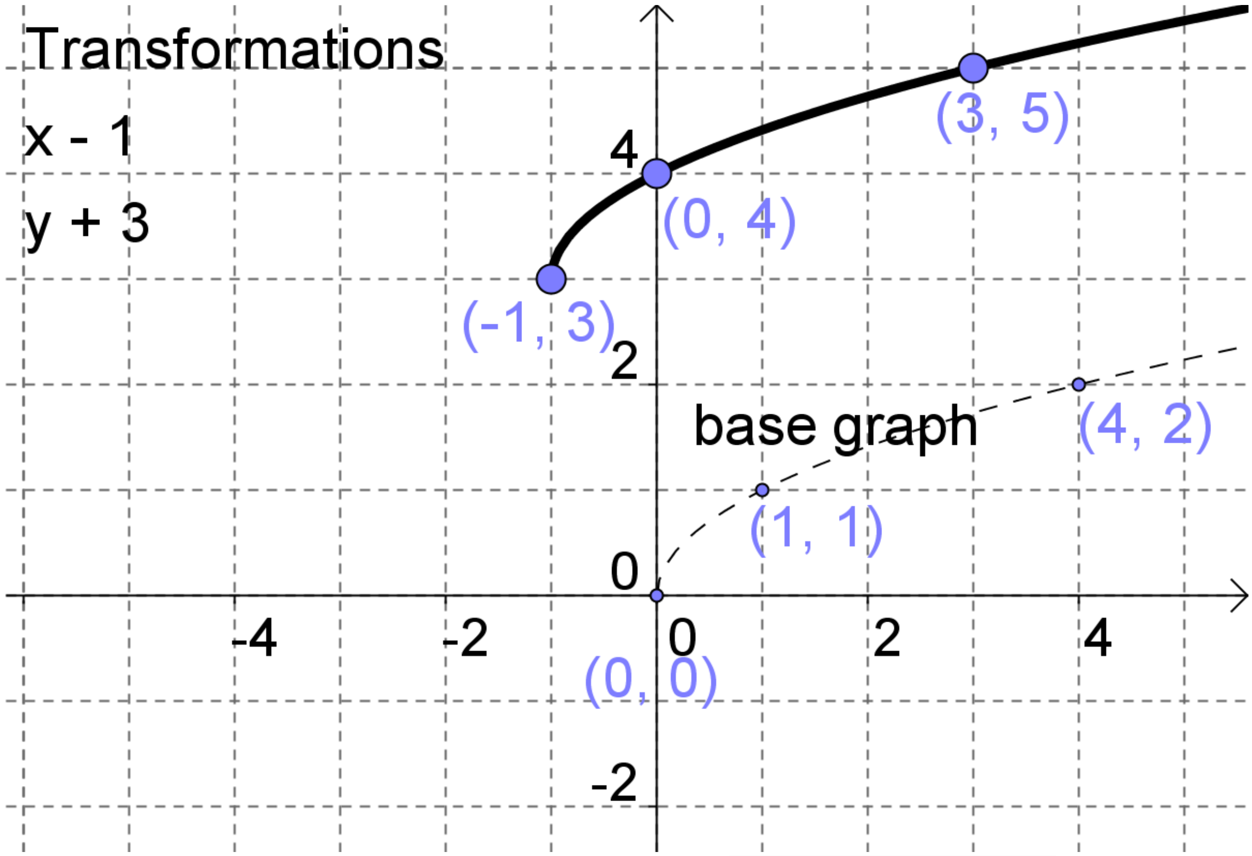
$x + 2 > 0$

$\Rightarrow \{x | x > -2\}$

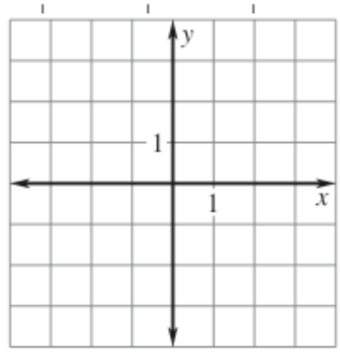
6. (3) Graph $f(x) = \frac{1}{2}|x - 3| + 5$



7. (3) Graph $f(x) = \sqrt{x+1} + 3$



8. (3) Graph the piecewise function $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$



x	y = 1/x
0	Und
-1	-1
-2	-1/2

1	1
4	2

9. (4) Determine the minimum of $f(x) = 3x^2 + 24x - 1$

The vertex is the minimum or maximum of any quadratic and is at $x = -\frac{b}{2a}$, then plug in that x-value in the equation to find the y-value

$$x = -\frac{b}{2a} = -\frac{24}{2 \cdot 3} = -\frac{24}{6} = -4$$

$$f(-4) = 3(-4)^2 + 24(-4) - 1 = 3 \cdot 16 - 96 - 1 = 48 - 96 - 1 = -48 - 1 = -49$$

$(-4, -49)$

10. (4) The price p (in dollars) and the quantity x sold of rubik's cubes obey the demand equation:

$$p = -\frac{1}{10}x + 150$$

a) Express the revenue R as a function of x. $R(x) = xp = x(\frac{-1}{10}x + 150) = \frac{-1}{10}x^2 + 150x$

b) What is the revenue if 100 units are sold? $R(10) = \frac{-1}{10}(10)^2 + 150(10) = -10 + 1500 = 1490$

c) What quantity x maximizes revenue? $\frac{-b}{2a} = \frac{-150}{2(-1/10)} = \frac{-150}{-1/5} = 750$

d) What is the maximum revenue? $R(750) = \frac{-1}{10}(750)^2 + 150(750) = \$56,250$

e) What price should the company charge to maximize revenue? $p = \frac{-1}{10}(750) + 150 = \$75$

11. (4), (11) Determine the equation of a parabola with vertex $(-5, -1)$ that goes through the point $(3, 1)$

The general equation is $f(x) = a(x - h)^2 + k$. The vertex tells us (h, k) and then we can plug in the other point to solve for a.

$f(x) = a(x + 5)^2 - 1$

$1 = a(3 + 5)^2 - 1$

$2 = a(8)^2$

$2 = 64a$

$a = \frac{1}{32}$

$\Rightarrow f(x) = \frac{1}{32}(x + 5)^2 - 1$

12. (6) Find the inverse function of $f(x) = 7x + 2$. Then check that you have the correct inverse by proving their compositions both equal x.

First we switch the x and y values, then we solve for y

$x = 7y + 2$

$x - 2 = 7y$

$\frac{1}{7}x - \frac{2}{7} = y$

$\Rightarrow f^{-1}(x) = \frac{1}{7}x - \frac{2}{7}$

$f(f^{-1}(x)) = 7(\frac{1}{7}x - \frac{2}{7}) + 2$ $\quad$ $f^{-1}(f(x)) = \frac{1}{7}(7x+2) - \frac{2}{7}$

$f(f^{-1}(x)) = (x-2) + 2$ $\quad$ $f^{-1}(f(x)) = x + \frac{2}{7} - \frac{2}{7}$

$f(f^{-1}(x)) = x$ $\quad$ $f(f^{-1}(x)) = x$

13. (5) Find each zero and its multiplicity then graph $f(x) = (x+3)^2(4x+3)(x^2-4)$.

Then find where f(x) < 0.

To find the zeros we set the y-value equal to zero and solve for x

$0 = (x+3)^2(4x+3)(x^2-4)$

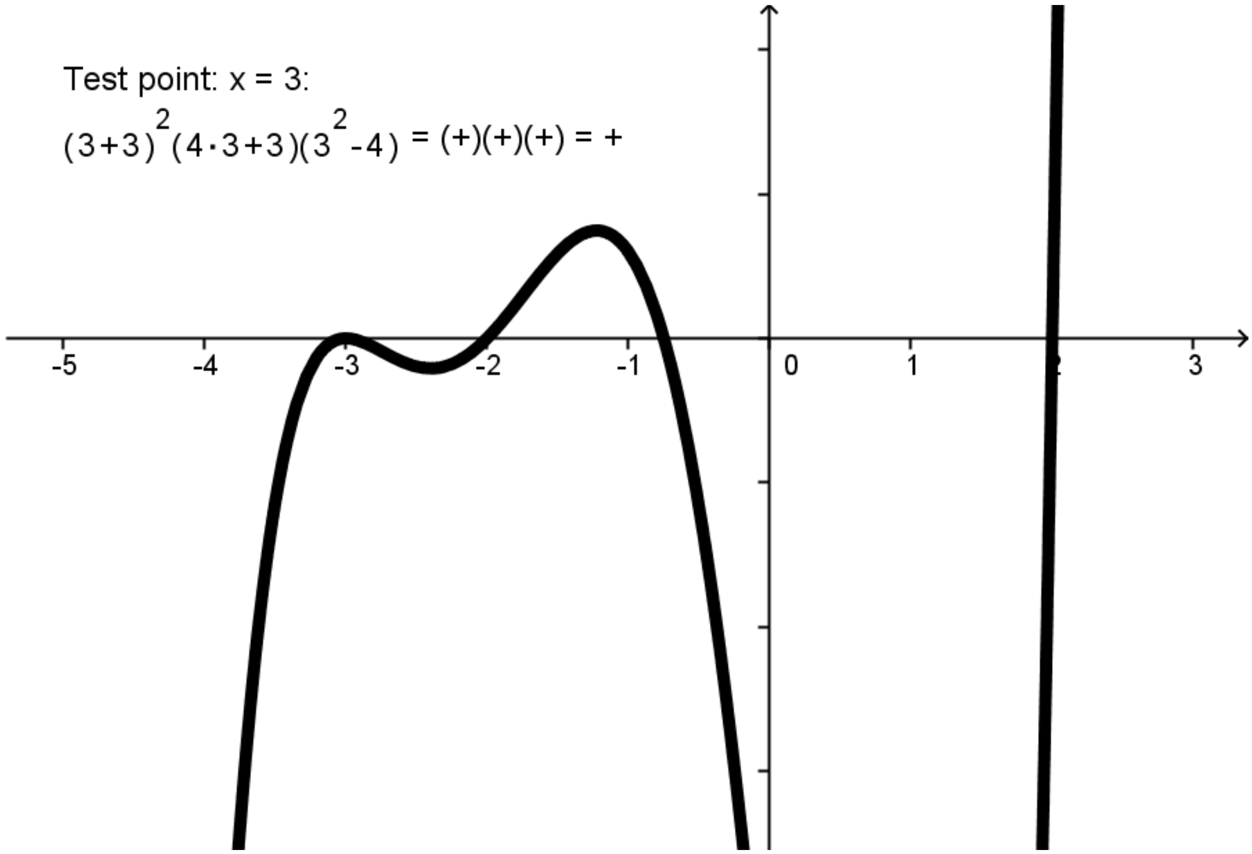
$0 = (x+3)^2; 0 = 4x+3; 0 = x^2 - 4$

$0 = x+3;\ -3 = 4x; 4 = x^2$

$-3 = x;\ -\frac{3}{4} = x;\ \pm 2 = x$

$x = -3\ (mult\ 2), -\frac{3}{4}(mult\ 1), 2\ (mult\ 1), -2\ (mult\ 1)$

To graph use the zeros along with odd multiplicities crossing and even multiplicities touching along with at least one test point



f(x) < 0 at (-inf, -3) U (-3, -2) U (-3/4, inf)

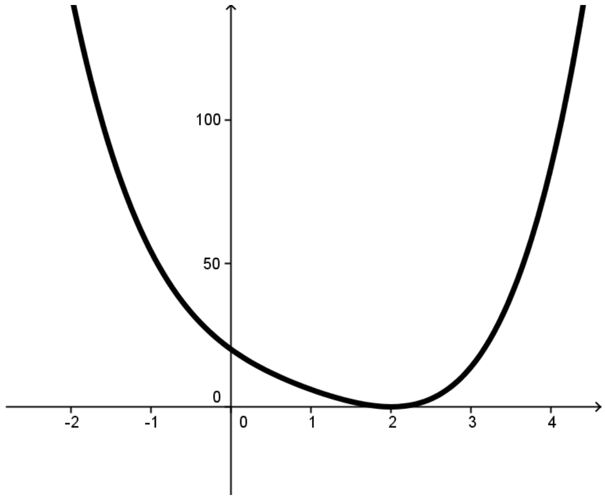
14. (5) Determine all of the possible rational zeros, find each zero and its multiplicity, and then graph

$f(x) = x^4 - 4x^3 + 9x^2 - 20x + 20$

To find the zeros we look at factors of the last term over factors of the first term

$$\pm\frac{1, 2, 4, 5, 10, 20}{1} = \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$$

To help us find what zero to attempt with the synthetic division we can look at its graph



This function appears to have a root at 2 with an even multiplicity:

2	1	-4	9	-20	20
		2	-4	10	-20
	1	-2	5	-10	0

Now we have factored $x^4 - 4x^3 + 9x^2 - 20x + 20 = (x-2)(x^3 - 2x^2 + 5x - 10)$ and we now have to solve $x^3 - 2x^2 + 5x - 10 = 0$, so we will continue with the synthetic division using the root of 2 again

2	1	-2	5	-10
		2	0	10
	1	0	5	0

Now we have factored $x^4 - 4x^3 + 9x^2 - 20x + 20 = (x-2)^2(x^2 + 5)$ and we need to solve

$x^2 + 5 = 0$

$x^2 + 5 = 0$

$x^2 = -5$

$x = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

$x = 2(mult\ 2); i\sqrt{5}(mult\ 1); -i\sqrt{5}(mult\ 1)$

I already showed the graph above

15. (5) Find each zero and its multiplicity, the y-intercept, the vertical asymptote(s), the horizontal asymptote, and then graph $f(x) = \frac{x+2}{x^2-9}$

Zeros will occur where the y value is 0:

$$0 = \frac{x+2}{x^2-9}$$

$$0 = x + 2$$

$$x = -2 \Rightarrow (-2, 0)$$

y-intercept will occur where the x value is 0:

$$y = \frac{0+2}{0^2-9} = \frac{2}{-9}$$

$$\Rightarrow \left(0, -\frac{2}{9}\right)$$

Vertical asymptotes will occur where the y-value goes to infinity, which happens when the bottom equals zero:

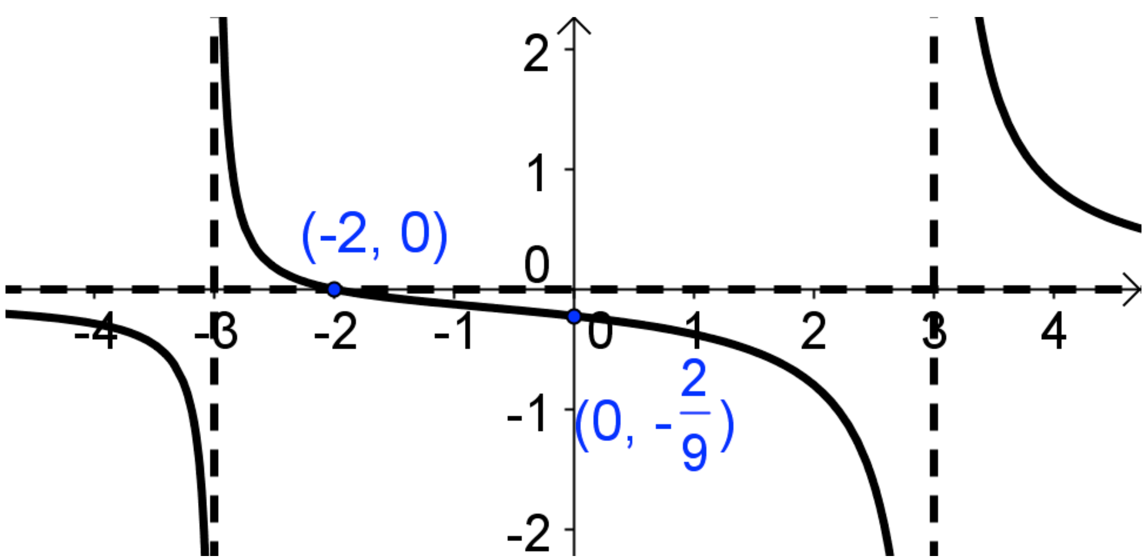
$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

Horizontal asymptote will occur where the x-value goes to infinity:

$$y = \lim_{x\to\infty} \frac{x+2}{x^2-9} \approx \frac{\infty+2}{\infty^2-9} \approx \frac{1}{\infty} = 0$$



16. (5) Write an equation of a polynomial in standard form with rational coefficients and the zeros 3, 2-i.

$f(x) = (x-3)(x-(2-i))(x-(2+i))$

$f(x) = (x-3)(x-2+i)(x-2-i)$

$f(x) = (x-3)(x^2-4x+5)$

$f(x) = x^3-7x^2+17x-15$

17. (5) Determine the remainder of $(4x^3-2x^2+1) \div (x+3)$

-3	4	-2	0	1
		-12	42	126
	4	-14	-42	127

Thus the remainder is 127

18. (5) Determine the maximum number of turning points of $f(x) = x^7+3x^3-2x^2+1$

Since the degree is 7, the maximum number of turning points (aka local max's and local min's) is 6

19. (6) Solve the equation $7^x = 20$

$\log_7 7^x = \log_7 20$

$x = \log_7 20 = \frac{\ln 20}{\ln 7} \approx 1.5395$

20. (6) Solve the equation $2\log_3(x+4) = 9$

$\frac{2\log_3(x+4)}{2} = \frac{9}{2}$

$\log_3(x+4) = \frac{9}{2}$

$3^{\log_3(x+4)} = 3^{\frac{9}{2}}$

$x+4 \approx 140.296$

$x \approx 136.296$

21. (6) Solve the equation $\log(x-2)+\log(x-3) = 2$

$\log\big((x-2)(x-3)\big) = 2$

$\log(x^2-5x+6) = 2$

$10^{\log(x^2-5x+6)} = 10^2$

$x^2 - 5x + 6 = 100$

$x^2 - 5x - 94 = 0$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-94)}}{2(1)} = \frac{5 \pm \sqrt{25+376}}{2} = \frac{5 \pm \sqrt{401}}{2}$$

$x \approx 12.51, -7.51$

Removing the extraneous solution we have the solution:

$\Longrightarrow x \approx 12.51$

22. (6) Rewrite $\ln\left(\frac{x^3y^2}{z^5}\right)$ using the sum or difference of logarithms with powers expressed as factors

$\ln x^3 + \ln y^2 - \ln z^5$

$= 3\ln x + 2\ln y - 5\ln z$

23. (6) Rewrite $2\log_3 x - \log_3 4 + 3\log_3(2y)$ as a single logarithm

$\log_3 x^2 - \log_3 4 + \log_3 (2y)^3$

$= \log_3 x^2 - \log_3 4 + \log_3 8y^3$

$= \log_3 \frac{8x^2y^3}{4}$

$= \log_3 2x^2y^3$

24. (6) Evaluate the logarithms without a calculator.

25. a.) $\log_4 256$ b.) $\log_2 \frac{1}{8}$

Logarithms ask us for the exponent that gets us from the base to the other number in the log.

a) 4 because 4^4 = 256 b) -3 because 2^-3 = 1/8

26. (6) If Bob deposits $300 at 3.7% interest compounded daily, how much will he have after two years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$= 300\left(1+\frac{0.037}{365}\right)^{365\cdot 2} = \$323.04$

27. (6) If Sally invests \$400 at 7% interest compounded continuously, how long will it take the account to reach \$700?

$$A = Pe^{rt}$$

$$700 = 400e^{0.07t}$$

$$\frac{700}{400} = \frac{400e^{0.07t}}{400}$$

$$\frac{7}{4} = e^{0.07t}$$

$$\ln\frac{7}{4} = \ln e^{0.07t}$$

$$\ln\frac{7}{4} = 0.07t$$

$$\frac{\ln\frac{7}{4}}{0.07} = t$$

$$t \approx 7.99\ years$$

28. (11) Find the vertex, focus, directrix, and then graph $(x+1)^2 = 8(y+1)$

$Vertex: (-1,-1)$

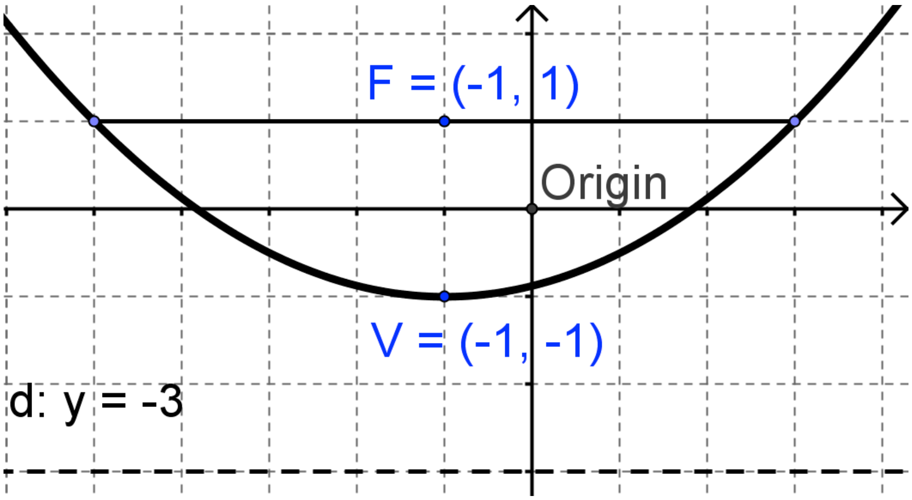
It opens up.

4a = 8 => a = 2

$Focus: (-1,-1+2) = (-1,1)$

$directrix: y = -1 - 2 \Rightarrow y = -3$

Length of Latus Rectum = 4a = 8



29. (11) Find the vertex, focus, directrix, and then graph $\frac{1}{2}(x-5)=(y-2)^2$

$Vertex: (5, 2)$

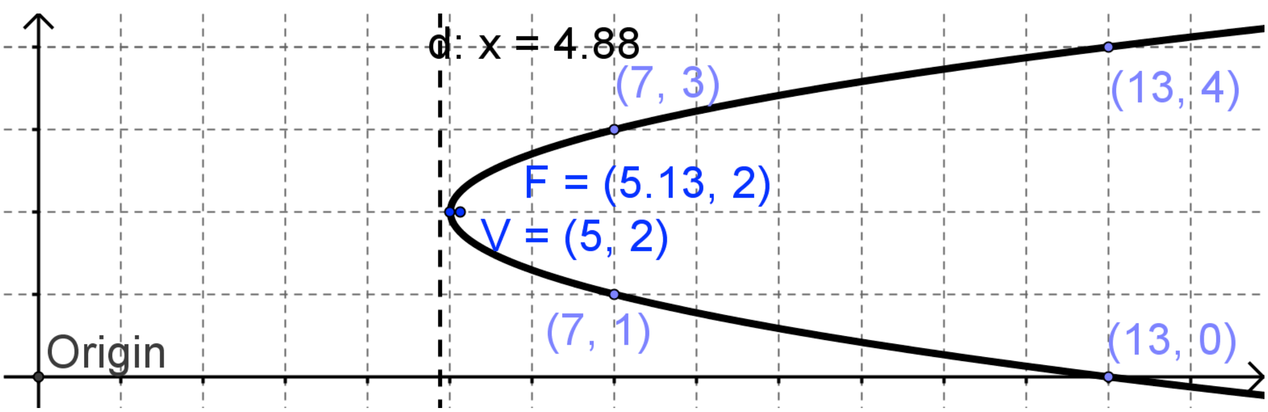
It opens right.

4a = ½ => a = 1/8

$Focus: \left(5+\frac{1}{8}, 2\right) = \left(5\frac{1}{8}, 2\right)$

$directrix: x = 5 - \frac{1}{8} \Rightarrow x = 4\frac{7}{8}$

Length of Latus Rectum = 4a = 1/2



30. (11) Put the equation into standard form and state the coordinates of the vertex $4y^2 - 3x - 8y - 5 = 0$

$4y^2 - 8y - 5 = 3x$

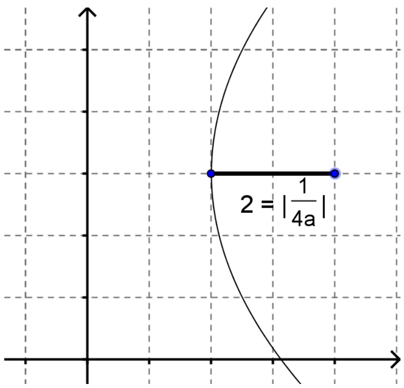
$4(y^2 - 2 + \mathbf{1}) - 5 - \mathbf{4} = 3x$

$4(y-1)^2 - 9 = 3x$

$$\frac{4}{3}(y-1)^2 - 3 = x$$

$$vertex: (-3, 1)$$

31. (11) Determine the equation of the parabola given a vertex of $(2, 3)$ and focus $(4, 3)$



When you sketch a graph you can see that it opens to the right, which means that it will be $x =$ with a positive a value

The coordinates of the vertex give us h and k

$$h = 2; k = 3$$

The distance between the vertex and the focus is 2, so we can set that equal to $\left|\frac{1}{4a}\right|$ and solve for a

$$2 = \left|\frac{1}{4a}\right|$$

$$2 = \frac{1}{4a}; \ -2 = \frac{1}{4a}$$

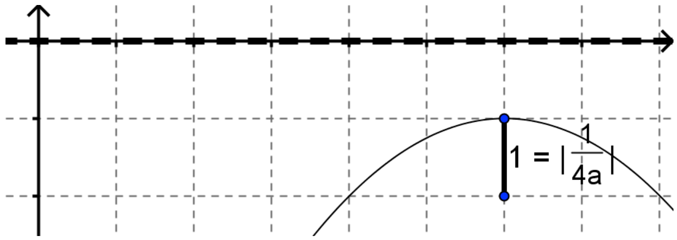
$$2a = \frac{1}{4}; \ -2a = \frac{1}{4}$$

$$a = \pm\frac{1}{8}$$

We know a has to be positive, so the final equation is

$$x = \frac{1}{8}(x-3)^2 + 2$$

32. (11) Determine the equation of the parabola given a focus of $(6, -2)$ and directrix $y = 0$



When you sketch a graph you can see that it opens down, which means that it will be $y =$ and a will be negative

The vertex has to be exactly halfway between the directrix and focus, so that gives us h and k

$h = 6; k = -1$

The distance between the vertex and focus is 1, so we set that equal to $\left|\frac{1}{4a}\right|$ to solve for a

$$1 = \left|\frac{1}{4a}\right|$$

$$1 = \frac{1}{4a}; -1 = \frac{1}{4a}$$

$$a = \pm\frac{1}{4}$$

We know a has to be negative, so the final equation is

$$y = -\frac{1}{4}(x-6)^2 - 1$$

33. (11) Find the center, foci, and vertices, then graph $\frac{(x-1)^2}{25} + \frac{(y+4)^2}{64} = 1$

$center: (1, -4)$

$vertices: (1, -4 \pm 8) \Rightarrow (1, 4)\ \&\ (1, -12)$

$f^2 = |a^2 - b^2|$

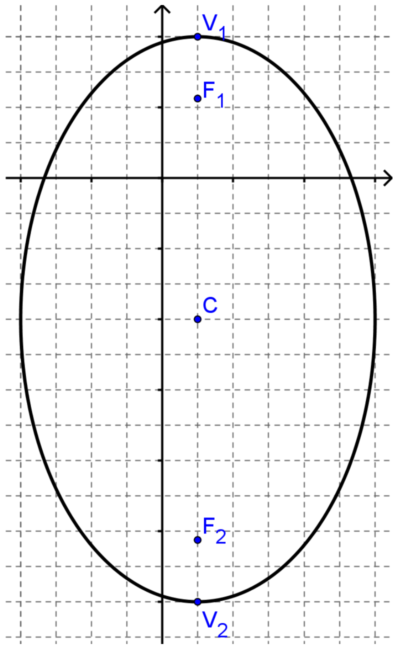
$f^2 = |25 - 64|$

$f^2 = |-39|$

$f^2 = 39$

$f = \sqrt{39}$

$foci: \left(1, -4 \pm \sqrt{39}\right)$



34. (11) Find the center, foci, and vertices, then graph $\frac{(y+2)^2}{9} - \frac{(x-3)^2}{16} = 1$

$center: (3, -2)$

$vertices: (3, -2 \pm 3) \Longrightarrow (3, 1)\ \&\ (3, -5)$

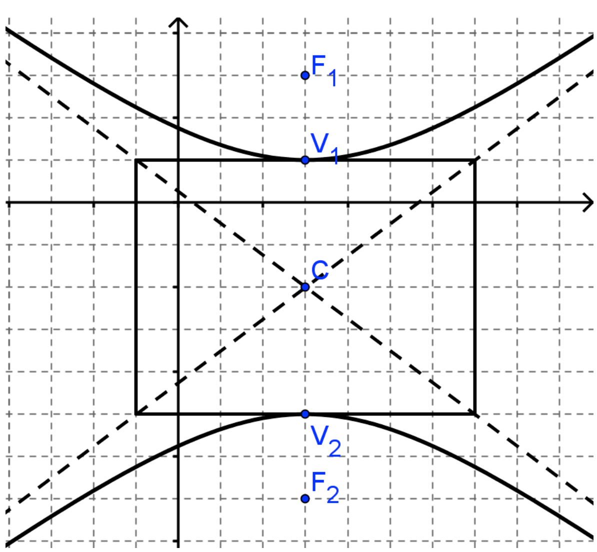
$f^2 = a^2 + b^2$

$f^2 = 9 + 16$

$f^2 = 25$

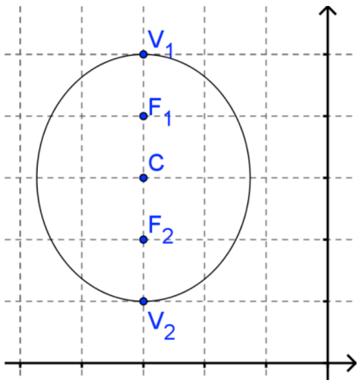
$f = 5$

$foci: (3, -2 \pm 5) \Longrightarrow (3, 3)\ \&\ (3, -7)$



35. (11) Determine the equation of the ellipse or hyperbola given foci $(-3,4)$ & $(-3,2)$ and vertices $(-3,5)$ & $(-3,1)$

Sketching a graph of the information we can tell it is an ellipse because the foci are inside the vertices. We can also tell that the major axis is vertical, so $b^2 > a^2$



The center has to be directly between the foci and vertices, so that tells us h and k

$h = -3; k = 3$

The distance from the center to the vertices tells us b

$b = 2$

The distance from the center to the foci tells us f, so we can use f and b to solve for a

$f = 1$

$f^2 = |a^2 - b^2|$

$1^2 = |a^2 - 2^2|$

$1 = a^2 - 4; -1 = a^2 - 4$

$5 = a^2; 3 = a^2$

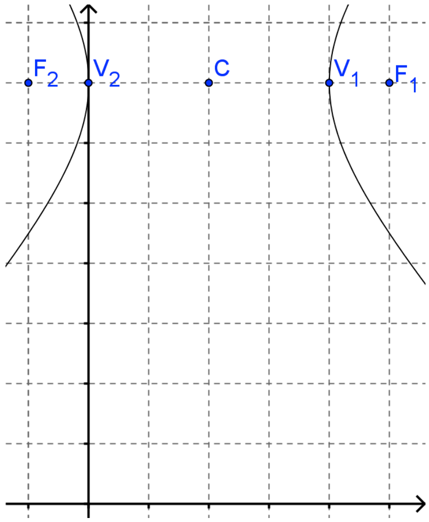
$a = \sqrt{5}, \sqrt{3}$

We know that $b^2 > a^2$, so a must be $\sqrt{3}$ and we can write the equation

$$\frac{(x+3)^2}{3} + \frac{(y-3)^2}{4} = 1$$

36. (11) Determine the equation of the ellipse or hyperbola given center $(2,7)$; focus $(5,7)$; and vertex $(4,7)$

Sketching a graph of the information we can tell it is a hyperbola because the foci are outside the vertices. We can also tell that the transverse axis is horizontal, so the x part is positive



The center tells us h and k

$h = 2; k = 7$

The distance from the center to the vertices tells us a

$a = 2$

The distance from the center to the foci tells us f, so we can use f and a to find b

$f = 3$

$f^2 = a^2 + b^2$

$3^2 = 2^2 + b^2$

$9 = 4 + b^2$

$5 = b^2$

$b = \sqrt{5}$

$$\frac{(x-2)^2}{4} - \frac{(y-k)^2}{5} = 1$$

37. (11) Put the equation in standard form and state the center for $4x^2 - 9y^2 + 40x - 72y - 19 = 0$

$4x^2 + 40x - 9y^2 - 72y = 19$

$4(x^2 + 10x + \mathbf{25}) - 9(y^2 + 8y + \mathbf{16}) - (\mathbf{4 \cdot 25 - 9 \cdot 16}) = 19$

$4(x+5)^2 - 9(y+4)^2 - -44 = 19$

$4(x+5)^2 - 9(y+4)^2 = -36$

$$\frac{4(x+5)^2}{-36} - \frac{9(y+4)^2}{-36} = \frac{-36}{-36}$$

$$\frac{(x+5)^2}{-9} - \frac{(y+4)^2}{-4} = 1$$

$$\frac{(y+4)^2}{4} - \frac{(x+5)^2}{9} = 1$$

$center: (-5, -4)$