Pre-Calc Exponential and Logarithmic Functions 6.1 – 6.8 Review

1. $f(x)=\sqrt{2x-5}$ $\qquad g(x)=3x^2-4$

a.) find $(f\circ g)(2)$

$f(g(2))$

$f(3(2)^2-4)$

$f(3\cdot 4-4)$

$f(8)$

$\sqrt{2(8)-5}$

$\sqrt{16-5}$

$\sqrt{11}$

b.) find $(g\circ g)(-3)$

$g(g(-3))$

$g(3(-3)^2-4)$

$g(3\cdot 9-4)$

$g(23)$

$3(23)^2-4$

$3\cdot 529-4$

$1587-4$

1583

c.) find $(g\circ f)(x)$

$g(f(x))$

check domain

$g(\sqrt{2x-5})$

$3(\cancel{\sqrt{2x-5}})^{\cancel{2}}-4$

$3(2x-5)-4$

$6x-15-4 = \boxed{6x-19}$

check domain

d.) Domain of $(g\circ f)(x)$

Domain:

$\{x \mid x \geq \frac{5}{2}\}$

$2x-5\geq 0$

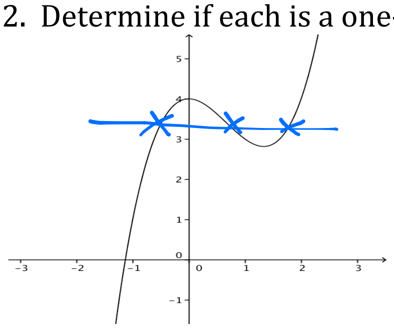
$2x\geq 5$

$x\geq \frac{5}{2}$

$6x-19$

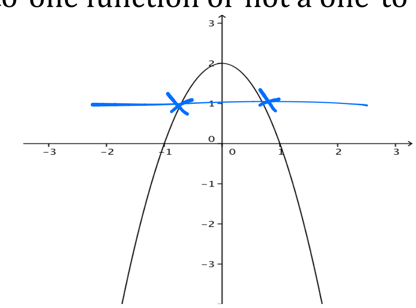
$x=\mathbb{R}$

2. Determine if each is a one-to-one function or not a one-to-one function.



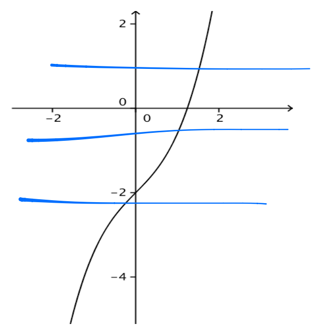
Function but NOT 1-1

It fails the Horizontal Line Test.



Function but NOT 1-1

" same "



Function and is 1-1

Passes the Horiz. Line Test

3. Determine the domain of $f(x)$, then find its inverse. Find the domain and range of both $f(x)$ and its inverse.

switch x & y

a.) $f(x) = \dfrac{x+4}{x-2}$

y

$(y-2)\cdot x = \dfrac{y+4}{\cancel{y-2}}\cdot\cancel{(y-2)}$

$\cancel{xy} - 2x = y + 4 - xy$

$-xy$

$-2x - 4 = y + \cancel{4} - xy$

-4

$-2x - 4 = y - xy$

$\dfrac{-2x-4}{(1-x)} = \dfrac{y\cancel{(1-x)}}{\cancel{(1-x)}}$

$f^{-1}(x) = \dfrac{-2x-4}{1-x}$

D of f : $\{x \mid x \neq 2\}$

R of f : $\{y \mid y \neq 1\}$

D of f^{-1} : $\{x \mid x \neq 1\}$

R of f^{-1} : $\{y \mid y \neq 2\}$

b.) $f(x) = \dfrac{x-3}{x+7}$

y

$(y+7)\cdot x = \dfrac{y-3}{\cancel{y+7}}\cdot\cancel{(y+7)}$

$\cancel{xy} + 7x = y - 3$

$-xy \qquad -xy$

$7x = y - xy - 3$

$+3 \qquad +3$

$7x + 3 = y - xy$

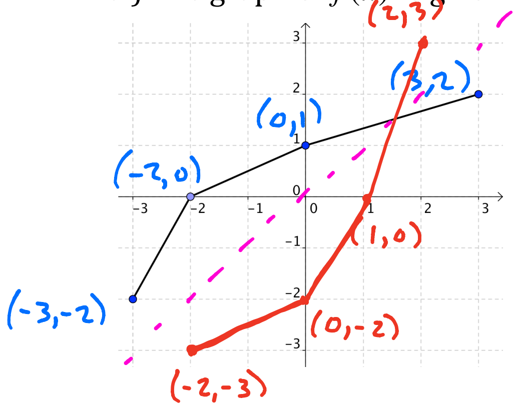
$\dfrac{7x+3}{1-x} = \dfrac{y\cancel{(1-x)}}{\cancel{1-x}}$

$f^{-1}(x) = \dfrac{7x+3}{1-x}$

D of f: $\{x \mid x \neq -7\}$ D of f^{-1}: $\{x \mid x \neq 1\}$

R of f: $\{y \mid y \neq 1\}$ R of f^{-1}: $\{y \mid y \neq -7\}$

4. a.) The graph of $f(x)$ is given. Graph y = x and $f^{-1}(x)$.



b.) Determine the exponential equation of the graph given two points: (-2 , -1/8) (1 , -8)

x y x y

$y = a(b)^x$

$-8 = a(b)^1$

$\dfrac{-8}{b} = \dfrac{a\cdot\cancel{b}}{\cancel{b}}$

$a = -\dfrac{8}{b}$

$y = -\dfrac{8}{b}(b)^x$

$-\dfrac{1}{8} = \dfrac{-8}{b}\cdot b^{-2}$

$-\dfrac{1}{8} = \dfrac{-8}{b}\cdot\dfrac{1}{b^2}$

$-\dfrac{1}{8} = \dfrac{-8}{b^3}$

$\dfrac{\cancel{-1}b^3}{\cancel{-1}} = \dfrac{-64}{-1}$

$\sqrt[3]{b^3} = \sqrt[3]{64}$

$b = 4$

$a = \dfrac{-8}{(4)}$

$a = -2$

Equation: $y = -2(4)^x$

5. Convert the logarithmic expression into an equivalent expression using an exponent.

a.) $\log_7 4 = y$

$7^y = 4$

b.) $\log_5(x+4) = 3$

$5^3 = x+4$

6. Convert exponential expression into an equivalent expression using a logarithm.

a.) $b^x = 5$

$\log_b 5 = x$

b.) $5^{x+4} = 8$

$\log_5 8 = x+4$

7. Determine the exact value

a.) $\log_4 256$

$4^x = 256$ $\boxed{\text{OR}}$ $\cancel{\log_4 4^4}$

$4^x = 4^4$

4

4

b.) $\log_2 \frac{1}{8}$

$2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

-3

8. Write the expression as a single logarithm. Express rational powers as roots.

a.) $2\log_2 b - \frac{1}{2}\log_2 5 + \log_2 7$

$[\log_2 b^2 - \log_2 5^{\frac{1}{2}}] + \log_2 7$

$\log_2\left(\frac{b^2}{\sqrt{5}}\right) + \log_2 7$

$\log_2 \frac{b^2}{\sqrt{5}} \cdot \frac{7}{1} = \log_2\left(\frac{7b^2}{\sqrt{5}}\right)$

b.) $\log_2 10 - \frac{1}{3}\log_2 8 - 3\log_2 v$

$[\log_2 10 - \log_2 8^{\frac{1}{3}}] - \log_2 v^3$

$\log_2\left(\frac{10}{\sqrt[3]{8}}\right) - \log_2 v^3$

$\log_2 \frac{10}{\sqrt[3]{8}\, v^3}$

9. Write the expression as the sum or difference of logarithms. Express powers or roots as factors.

a.) $\ln\left(\frac{a^3c^2}{d^4}\right)$ Division

$\ln a^3c^2 - \ln d^4$

$\ln a^3 + \ln c^2 - \ln d^4$

$3\ln a + 2\ln c - 4\ln d$

b.) $\log_4 \frac{\sqrt{x-2}}{x+1}$ Division

$\log_4 \sqrt{x-2} - \log_4 (x+1)$

$\log_4 (x-2)^{\frac{1}{2}} - \log_4 (x+1)$

$\frac{1}{2}\log_4 (x-2) - \log_4 (x+1)$

10. Solve each equation for all real solutions. Express in exact form.

a.) $3^{-2x} = (27)^{x-2}$

$$3^{-2x} = 3^{3(x-2)}$$

$$-2x = 3x - 6$$

$$-3x \quad -3x$$

$$\frac{-5x}{-5} = \frac{-6}{-5}$$

$$x = \frac{6}{5}$$

b.) $(\log_4 x + \log_4(x+3)) = 1$

$$\log_4 x(x+3) = 1$$

Rewrite:

$$4^1 = x(x+3)$$

$$4 = x^2 + 3x$$

$$0 = x^2 + 3x - 4$$

$$0 = (x+4)(x-1)$$

$$\cancel{x = -4} \qquad x = 1$$

Extraneous Solution

$\log_4(-4)$ can't be Negative

c.) $\ln 6 = \ln 4^{4x}$

$$\frac{\ln 6}{\ln 4} = \frac{4x \ln 4}{\ln 4}$$

$$\frac{\frac{\ln 6}{\ln 4}}{4} = \frac{4x}{4}$$

EXACT VALUE $x = \frac{\frac{\ln 6}{\ln 4}}{4}$ or $\frac{\frac{\log 6}{\log 4}}{4}$

OR $\log_4 6 = 4x$

$$\frac{\log_4 6}{4} = x$$

APPROXIMATE VALUE $x \approx .32$

d.) $\ln(10x) = \ln(x-2) + 1$

$$-\ln(x-2) \quad -\ln(x-2)$$

$$\ln(10x) - \ln(x-2) = 1$$

$$\ln\left(\frac{10x}{x-2}\right) = 1$$

$$\log_e\left(\frac{10x}{x-2}\right) = 1$$

$$(x-2)\cdot e^1 = \frac{10x}{x-2}\cdot(x-2)$$

$$ex - 2e = 10x$$

$$-ex \qquad -ex$$

$$-2e = 10x - ex$$

$$\frac{-2e}{(10-e)} = \frac{x(10-e)}{(10-e)}$$

tough! → $x = \frac{-2e}{10-e} \approx -.7466$

11. Evaluate each logarithm. Round to 4 decimal places.

a.) $\log_4 12$

CHANGE OF BASE FORMULA.

Calculator:

$$\frac{\log 12}{\log 4} \approx 1.79$$

b.) $\log_{1.3} \sqrt{14}$

Calculator:

$$\frac{\log \sqrt{14}}{\log 1.3} \approx 5.03$$

or

$$\frac{\log 14^{\frac{1}{2}}}{\log 1.3} \approx 5.03$$

12. Solve each exponential equation. Express as exact and approximate answers rounded to 3 decimal places.

a.) $\ln 5^{x} = \ln 3^{x+2}$

$x \ln 5 = (x+2) \ln 3$

$x \ln 5 = \cancel{x \ln 3} + 2 \ln 3$

$-x \ln 3 \quad -\cancel{x \ln 3}$

$x \ln 5 - x \ln 3 = 2 \ln 3$

$x(\ln 5 - \ln 3) = \ln 3^{2}$

$\frac{x \cancel{\left(\ln \frac{5}{3}\right)}}{\cancel{\ln\left(\frac{5}{3}\right)}} = \frac{\ln 9}{\ln\left(\frac{5}{3}\right)}$

$x = \frac{\ln 9}{\ln\left(\frac{5}{3}\right)} \approx 4.30$

b.) $\ln 5^{x+2} = \ln 7^{x-2}$

$(x+2) \ln 5 = (x-2) \ln 7$

$x \ln 5 + 2 \ln 5 = \cancel{x \ln 7} - 2 \ln 7$

$-x \ln 7 \quad -\cancel{x \ln 7}$

$x \ln 5 - x \ln 7 + \cancel{2 \ln 5} = -2 \ln 7$

$-\cancel{2 \ln 5} \quad -2 \ln 5$

$x(\ln 5 - \ln 7) = -2 \ln 7 - 2 \ln 5$

$x\left(\ln \frac{5}{7}\right) = \ln 7^{-2} + \ln 5^{-2}$

$x\left(\ln \frac{5}{7}\right) = \ln \frac{1}{49} + \ln \frac{1}{25}$

$x\left(\ln \frac{5}{7}\right) = \ln\left(\frac{1}{49 \cdot 25}\right)$

$\frac{x \cancel{\left(\ln \frac{5}{7}\right)}}{\cancel{\ln\left(\frac{5}{7}\right)}} = \frac{\ln\left(\frac{1}{1225}\right)}{\ln\left(\frac{5}{7}\right)}$

$x = \frac{\ln\left(\frac{1}{1225}\right)}{\ln\left(\frac{5}{7}\right)} \approx 21.13$

13.Find the value of $200 invested at 10% interest compounded quarterly for 8 years.

$A = 200\left(1 + \frac{.10}{4}\right)^{(4 \cdot 8)}$

$= 200\,(1.025)^{(32)}$

$= \$440.75$

14. How long will it take for $200 to grow to $1,500 with a continuously compounded rate of 6.2%?

$$\frac{15\cancel{00}}{2\cancel{00}} = \frac{\cancel{200}e^{(.062t)}}{\cancel{200}}$$

$$\ln\frac{15}{2} = \cancel{\ln} e^{.062t}$$

$$\frac{\ln\left(\frac{15}{2}\right)}{.062} = \frac{\cancel{.062}t}{\cancel{.062}}$$

$$t = 32.5 \text{ years}$$

15. What is the initial investment required if an account grows to $3,000 at 5.3% interest compounded monthly in 10 years?

$$3000 = P\left(1+\frac{.053}{12}\right)^{(12\cdot 10)}$$

$$\frac{3000}{(1.0044)^{120}} = \frac{P\cancel{(1.0044)^{120}}}{\cancel{(1.0044)^{120}}}$$

$$P = \$1767.88$$

16. How long does it take for $100 to triple in value if the interest is compounded weekly at 6% APR.?

$$\frac{3\cancel{00}}{1\cancel{00}} = \frac{\cancel{100}\left(1+\frac{.06}{52}\right)^{(52\cdot t)}}{\cancel{100}}$$

$$3 = (1.00115)^{52t}$$

$$\frac{\log_{1.00115}(3)}{52} = \frac{\cancel{52}t}{\cancel{52}}$$

$$\frac{\frac{\log 3}{\log 1.00115}}{52} = t$$

$$18.38 \text{ years} = t$$

The number of bacteria on a countertop after t hours is given by the equation
$N = 950e^{0.04t}$

17. How many bacteria are present after 6 hours?

$950e^{(.04 \times 6)} = 1207.69$ bacteria

18. How long will it take for the count to be 5,000 bacteria?

$\frac{5000}{950} = \frac{\cancel{950}e^{.04t}}{\cancel{950}}$

$\ln 5.26 = \cancel{\ln e}^{.04t}$

$\frac{\ln 5.26}{.04} = \frac{\cancel{.04}t}{\cancel{.04}}$

$t = 41.52$ hours

There were 900 Polar Bears in the wild in 1972. In 1997, there were 2045 bears.

19. Write an equation for the number of bears t years after 1972 assuming uninhibited growth.

$\frac{2045}{900} = \frac{\cancel{900}e^{(k \cdot 25)}}{\cancel{900}}$

$\ln \frac{2045}{900} = \cancel{\ln e}^{25k}$

$\frac{\ln \frac{2045}{900}}{25} = \frac{\cancel{25}k}{\cancel{25}}$

$k = \frac{\ln \frac{2045}{900}}{25}$ or $.0328$

$N(t) = 900e^{(.0328t)}$

20. Use the equation found in question 11 to estimate the number of Polar Bears in 2015.

That's 43 years after 1972. $= 900e^{(.0328 \times 43)}$

$\approx$ 3,688 polar bears in 2015