6.7/6.8 Notes

Simple Interest	Compound Interest	Continuously Compounded Interest
$I = Prt$	$A = P\left(1+\frac{r}{n}\right)^{nt}$	$A = Pe^{rt}$
I= Interest Charged r= interest Rate t= time (in years)	P= Principal (\$) r= interest Rate n= number of times Compounded t= time (years)	P= Principal (\$) r= rate (interest) t= time (years)

1. Find the amount that results from each investment after 10 years:

a. \$5,000 invested at 4.2% compounded **annually**.

.042 n=1

$5000\left(1+\frac{.042}{1}\right)^{(1\cdot 10)}$

\$7,544.79

b. \$5,000 invested at 4.2% compounded **monthly**.

n=12

$5000\left(1+\frac{.042}{12}\right)^{(12\cdot 10)}$

\$7,604.23

c. \$5,000 invested at 4.2% compounded **daily**.

n=365

$5000\left(1+\frac{.042}{365}\right)^{(365\cdot 10)}$

\$7,609.62

d. \$5,000 invested at 4.2% compounded **continuously**.

n= ?

$5000e^{(.042\cdot 10)}$

Pe^{rt}

\$7609.81

2. Find the present value needed to get \$2,000 after 4 years at 5% compounded monthly.

$A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$

$2000 = P\left(1+\frac{.05}{12}\right)^{(12\cdot 4)}$

$\frac{2000}{(1.004167)^{48}} = \frac{P(1.004167)^{48}}{(1.004167)^{48}}$

P= \$1638.14

3. Find the present value needed to get \$2,000 after 4 years at 5% compounded continuously.

$A = Pe^{rt}$

$2000 = Pe^{(.05\cdot 4)}$

$\frac{2000}{e^{.2}} = \frac{Pe^{.2}}{e^{.2}}$

P= \$1637.46

4. Austin will be buying a used car for \$12,000 in 3 years. How much money should he ask his parents for now so that, if he invests it at 4.5% compounded continuously, he will have enough to buy the car?

$12000 = Pe^{(.045\cdot 3)}$

$\frac{12000}{e^{.135}} = \frac{Pe^{(.135)}}{e^{.135}}$

P= \$10,484.59

5. How many years will it take for an initial investment of \$20,000 to grow to \$50,000? Assume a rate of interest of 6% compounded continuously.

$\frac{50,000}{20,000} = \frac{20,000e^{(.06t)}}{20,000}$

$\ln\frac{5}{2} = \ln e^{.06t}$

$\frac{\ln\frac{5}{2}}{.06} = \frac{.06t}{.06}$

t = 15.27 years

6. How long will it take for an investment to **double** if it earns 5% compounded continuously?

$A = Pe^{rt}$

$\frac{2}{1} = \frac{1e^{(.05t)}}{1}$

$\ln 2 = \ln e^{.05t}$

$\frac{\ln 2}{.05} = \frac{.05t}{.05}$

t = 13.86 years

7. How long will it take for an investment to **triple** if it earns 5% compounded continuously?

$\ln 3 = \ln e^{(.05t)}$

$\frac{\ln 3}{.05} = \frac{.05t}{.05}$

t = 21.97 years

$N(t) = N_0 e^{kt}$

Sir Isaac Newton, co-founder of Calculus

Law of Uninhibited Growth	Newton's Law of Cooling	
$A(t) = A_0 e^{kt}$	$u(t) = T + (u_0 - T)e^{kt}$	
A_0= initial Amount of Population	u(t)= Temp. after "t" min.	
k= Growth Rate (positive value)	T= Constant Temp.	k= decay rate (negative constant)
t= time (must be specified)	u_0= initial Temp. of a heated object	t= time

8. The size N of an ant population at time *t* (in days) obeys the function: $N(t) = 800e^{0.034t}$

a. Determine the number of ants at t = 0.

800 ants

b. What is the growth rate of the ants population?

$.034 \longrightarrow 3.4\%$

c. What is the population after 15 days?

$800e^{(.034 \times 15)}$

≈ 1332 ants

d. When will the ant population reach 2,000?

$\frac{2{,}000}{800} = \frac{800e^{(.034t)}}{800}$

$\ln\frac{5}{2} = \cancel{\ln e}^{.034t}$

$\frac{\ln\frac{5}{2}}{.034} = \frac{.034t}{.034}$

$t \approx 27$ days

26.9

e. When will the ant population triple?

$\frac{2400}{800} = \frac{800e^{(.034t)}}{800}$

$\ln 3\ \cancel{\ln e}^{.034t}$

$\frac{\ln 3}{.034} = \frac{.034t}{.034}$

$t \approx 32$ days

9. The population of Portland, Oregon follows the exponential law.

a. If N is the population of the city and *t* is the time in years, express N as a function of *t*.

$N(t) = N_0 e^{kt}$

b. From the years 2000 to 2010, the population of Portland increased from 529,000 to 584,000, respectively. Write an equation assuming uninhibited growth.

$\boxed{N(t) = 529{,}000\, e^{\left(\frac{\ln\frac{584}{529}}{10} \cdot t\right)}}$

$\frac{584{,}000}{529{,}000} = \frac{529{,}000e^{(k \cdot 10)}}{529{,}000}$

$\ln\frac{584}{529} = \cancel{\ln e}^{10k}$

$\frac{\ln\frac{584}{529}}{10} = \frac{10k}{10}$

$k = \frac{\ln\frac{584}{529}}{10}$

$k = .00989$

c. What will the population be in 2020?

$N(t) = 529{,}000\, e^{\left(\frac{\ln\frac{584}{529}}{10} \cdot 20\right)}$

$N(20) \approx 644{,}718$ people

10. An object is heated to 100° C and is then allowed to cool in a room whose air temperature is 30° C.

a. If the temperature of the object is 80° C after 5 minutes, write an equation using Newton's Law of Cooling.

$80 = 30 + (100-30)^{(k \cdot 5)}$

$80 - 30 = 30 - 30 + (70)^{5k}$

$\log_{70} 50 = \cancel{\log_{70}}(70)^{5k}$

$\frac{\log_{70} 50}{5} = \frac{5k}{5}$

$= k$

$u(t) = 30 + (70)^{\left(\frac{\log_{70} 50}{5} \cdot t\right)}$

.18416

or

$u(t) = 30 + (70)^{.18416t}$

b. When will its temperature be 50° C?

$50 = 30 + (70)^{\left(\frac{\log_{70} 50}{5} \cdot t\right)}$

$-30 \quad -30$

$20 = 70^{.18416t}$

$\frac{\log_{70} 20}{.18416} = \frac{.18416t}{.18416}$

$t = 3.8$ minutes