

Evaluate each expression. Give exact answers! Keep any angle measures in radians.

1. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\theta \text{ to } \pi$

$$x = -\frac{\sqrt{3}}{2} ?$$

$$\theta = \frac{5\pi}{6}$$

from $\frac{\pi}{2}$

2. $\csc^{-1}(\sqrt{2})$

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\theta = \frac{\pi}{4}$$

3. $\tan^{-1}(-1)$

$$\frac{y}{x} = -1 \quad ?$$

$$\text{at } \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\theta = -\frac{\pi}{4}$$

NOT $\frac{7\pi}{4}$

4. $\sin\left(\frac{5\pi}{6}\right)$

y-value at $\frac{5\pi}{6}$

$$\frac{1}{2}$$

5. $\cos^{-1}(-4)$ Not on unit circle!

$$\begin{array}{|c|c|} \hline 1 & -4 \\ \hline 1^2 = (-4)^2 + y^2 & \\ 1 = 16 + y^2 & \\ y^2 = -15 & \text{No solution!} \\ \hline \end{array}$$

Undefined

6. $\sec^{-1}(2)$

$$\cos^{-1}\left(\frac{1}{2}\right)$$

$$x = \frac{1}{2} ?$$

$$\theta = \frac{\pi}{3}$$

7. $\sec\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$$\sec\left(\frac{\pi}{3}\right)$$

flip $\cos\left(\frac{\pi}{3}\right)$

$$= \frac{1}{2}$$

8. $\csc\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$$\csc\left(\frac{5\pi}{6}\right)$$

flip sin

$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

9. $\sec\left(\tan^{-1}(\sqrt{3})\right)$

$$\sec\left(\frac{\pi}{3}\right)$$
 flip cos

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\theta = 2$$

10. $\csc\left(\cos^{-1}\left(-\frac{3}{8}\right)\right)$ Not on unit circle!

$$\begin{array}{|c|c|} \hline 8 & -3 \\ \hline 64 = 9 + y^2 & \\ \sqrt{55} = \sqrt{y^2} & \\ \hline \end{array}$$

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12. $\csc\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) + \cot\left(\tan^{-1}(1)\right)$

$$\csc\left(\frac{\pi}{6}\right) + \cot\left(\frac{\pi}{4}\right) \rightarrow \text{flip}$$

$$\frac{2}{1} + 1$$

$$\tan\frac{\pi}{4} = \frac{1}{1}$$

$$\theta = 3$$

13. $\sin^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) + \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

$$-\frac{3\pi}{4} + \frac{3\pi}{4} = \frac{2\pi}{4}$$

$$\theta = \frac{\pi}{2}$$

Solve the equation. Give the general formula for all solutions! Show all work! Circle final answers!

14. $6\tan\theta + 13 = 19$

$$\frac{6\tan\theta}{6} = \frac{-13}{6}$$

$$\tan\theta = 1$$

$$\theta = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\} + \pi k$$

15. $\sin(2\theta) - \frac{\sqrt{3}}{2} = 0$

$$\sin(2\theta) = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2} \cdot 2\theta = \left\{ \frac{\pi}{3}, \frac{1}{2} \cdot \frac{2\pi}{3}, \frac{1}{2} \right\} \pm \pi k \cdot \frac{1}{2}$$

$$\frac{2\pi}{6} = \frac{\pi}{3}$$

$$\theta = \left\{ \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \right\} + \pi k$$

$$+ \frac{6\pi}{6} \quad + \frac{3\pi}{3}$$

Solve the equation over the interval $0 \leq \theta < 2\pi$. Give exact answers, show all work, and circle final answers!

18. $\frac{2\sin(2\theta)}{2} = -\frac{\sqrt{3}}{2}$

$$\sin(2\theta) = -\frac{\sqrt{3}}{2}$$

$$\frac{1}{2} \cdot 2\theta = \left\{ \frac{4\pi}{3}, \frac{5\pi}{2}, \frac{2\pi}{3}, \frac{13}{2} \right\} \pm 2\pi k \cdot \frac{1}{2}$$

$$\theta = \left\{ \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{10\pi}{6}, \frac{11\pi}{6} \right\} + \pi k$$

$$+ \frac{6\pi}{6} \quad + \frac{6\pi}{6}$$

$$\theta = \left\{ \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6} \right\}$$

20. $\sin(2\theta) = -\cos\theta$

$$2\sin\theta\cos\theta = -\cos\theta$$

$$+ \cos\theta + \cos\theta$$

$$2\sin\theta\cos\theta + \cos\theta = 0$$

$$\cos\theta(2\sin\theta + 1) = 0$$

$$\cos\theta = 0 \quad 2\sin\theta + 1 = 0$$

$$\sin\theta = \frac{-1}{2}$$

19. $2\cos^2\theta + 9\cos\theta - 5 = 0$

$$(2\cos\theta - 1)(\cos\theta + 5) = 0$$

$$2\cos\theta - 1 = 0$$

$$\cos\theta + 5 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$$\cos\theta = -\frac{5}{1}$$

$$1^2 = (-5)^2 + y^2$$

$$1 = 25 + y^2$$

$$\sqrt{-24} = \sqrt{y^2}$$

undefined

to factor, need equation
equal to 0!

can't divide
by $\cos\theta$!

$$\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$