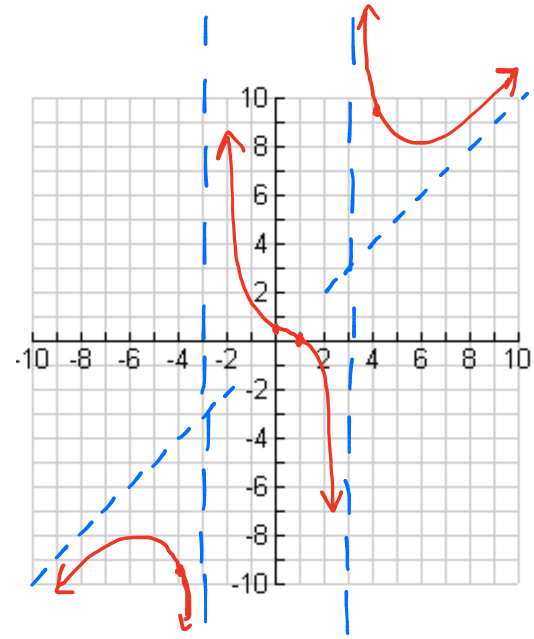
Precal chapter 5 Rational Functions Reviews



1. $R(x) = \dfrac{x^3 - 1}{x^2 - 9}$ $\qquad \dfrac{x^3 - 1}{(x+3)(x-3)}$

a) Domain: $\{x \mid x \neq 3, -3\}$

b) Vertical Asymptote(s): $x = 3$ $\quad x = -3$

c) x-intercept(s): $(1, 0)$

$0 = x^3 - 1$

$1 = x^3$

$\sqrt[3]{1} = \sqrt[3]{x^3}$

$1 = x$

d) y-intercepts(s): $(0, \frac{1}{9})$ $\quad \frac{-1}{-9}$

e) End Behavior Asymptote: $y = x$

$$\begin{array}{r} x + \frac{9x-1}{x^2-9} \\ x^2+0x-9 \overline{)\, x^3 + 0x^2 + 0x - 1} \\ +(-x^3 - 0x^2 + 9x) \\ \hline 9x - 1 \;\; R \end{array}$$

f) Graph the function.

g) Limits of the ends and near each vertical asymptote:

$\lim_{x \to -\infty} f(x) = -\infty$ $\qquad \lim_{x \to +\infty} f(x) = \infty$

$\lim_{x \to -3^-} f(x) = -\infty$ $\qquad \lim_{x \to -3^+} f(x) = \infty$

$\lim_{x \to 3^-} f(x) = -\infty$ $\qquad \lim_{x \to 3^+} f(x) = \infty$

Test $x = -4$

$\dfrac{(-4)^3 - 1}{(-4+3)(-4-3)}$

$\dfrac{-64-1}{(-1)(-7)} = \dfrac{-65}{7}$

Test $x = 4$

$\dfrac{(4)^3 - 1}{(4+3)(4-3)}$

$\dfrac{64-1}{(7)(1)} = \dfrac{63}{7}$

2. $g(x) = \frac{3x+6}{x-5}$ $\frac{3(x+2)}{x-5}$

a) Domain: $\{x \mid x \neq 5\}$

b) Vertical Asymptote(s): $x = 5$

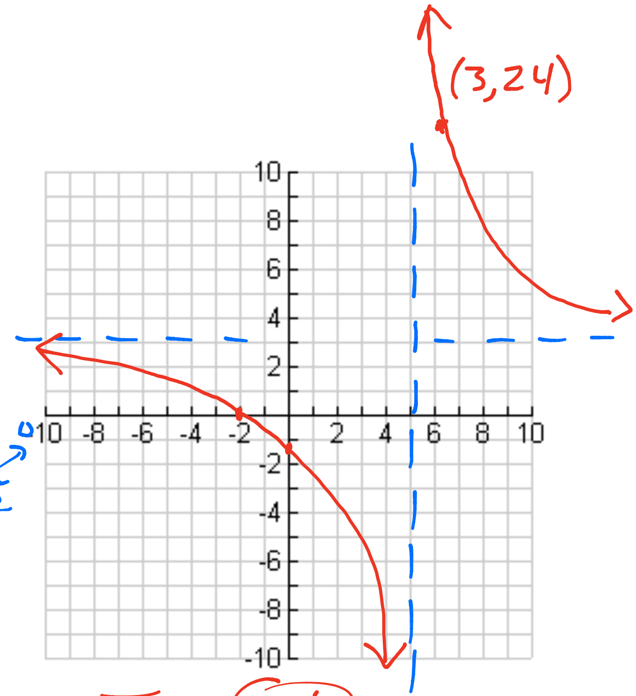
c) x-intercept(s): $(-2, 0)$ $0 = 3(x+2)$, $0 = x+2$, $x = -2$

d) y-intercepts(s): $(0, -\frac{6}{5})$

e) End Behavior Asymptote: $y = 3$ $\frac{3x}{x}$

$x-5 \overline{)3x+6}$ → $3 + \frac{21}{x-5}$; $-(3x-15)$; 21

f) Graph the function.



g) Limits of the ends and near each vertical asymptote:

Test $x = 6$: $\frac{3(6+2)}{6-5} = \frac{3(8)}{1} = 24$

$\lim_{x \to -\infty} f(x) = 3$ $\lim_{x \to +\infty} f(x) = 3$

$\lim_{x \to 5^-} f(x) = -\infty$ $\lim_{x \to 5^+} f(x) = \infty$

3. $h(x) = \frac{x-2}{2x^2-8}$ $\frac{x-2}{2(x^2-4)} = \frac{x-2}{2(x+2)(x-2)}$ $\frac{1}{2(x+2)}$

a) Domain: $\{x \mid x \neq 2, -2\}$ Hole at $x = 2$

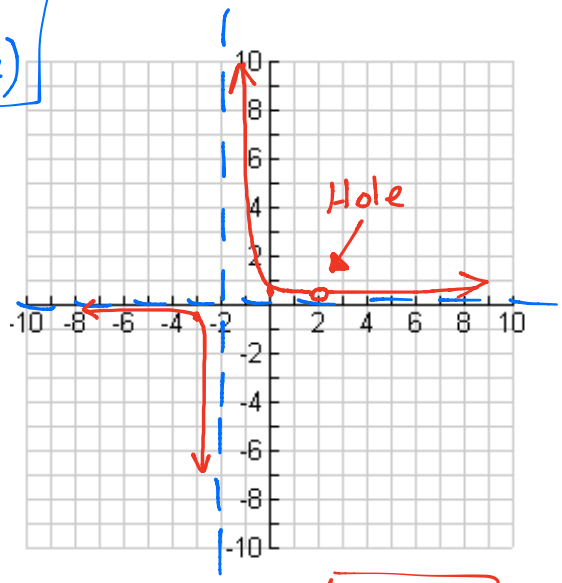
b) Vertical Asymptote(s): $x = -2$

c) x-intercept(s): NONE $0 \neq 1$ $0 = \frac{1}{2(x+2)}$

d) y-intercepts(s): $(0, \frac{1}{4})$ $\frac{1}{2(0+2)} = \frac{1}{2(2)}$

e) End Behavior Asymptote: $y = 0$ $\frac{1}{2(x+2)} \to 0$

f) Graph the function.



g) Limits of the ends and near each vertical asymptote:

$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to +\infty} f(x) = 0$

$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = \infty$

Test $x = -3$

$\frac{1}{2(-3+2)}$

$\frac{1}{2(-1)}$

$-\frac{1}{2}$

4. $p(x) = \dfrac{x^2 + x - 12}{x - 4}$ $\dfrac{(x+4)(x-3)}{x-4}$

a) Domain: $\{x \mid x \neq 4\}$

b) Vertical Asymptote(s): $x = 4$

c) x-intercept(s): $(-4,0)(3,0)$ $0 = (x+4)(x-3)$

d) y-intercepts(s): $(0, 3)$

$\frac{-12}{-4}$

e) End Behavior Asymptote: $y = x + 5$

f) Graph the function.

$x-4 \overline{)\, x^2 + x - 12}$

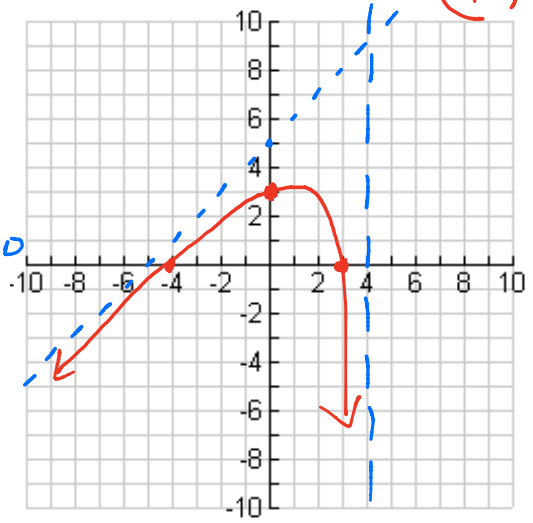
$x + 5 + \frac{8}{x-4}$ → 0

$-(x^2 + 4x)$

$5x - 12$

$-(5x + 20)$

-8



(6,15)

g) Limits of the ends and near each vertical asymptote:

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to +\infty} f(x) = \infty$

$\lim_{x \to 4^-} f(x) = -\infty$ $\lim_{x \to 4^+} f(x) = \infty$

Test $x = 6$

$\dfrac{(6+4)(6-3)}{(6-4)}$

$\dfrac{(10)(3)}{2}$

$\frac{30}{2}$

15

5. $f(x)=\dfrac{x+1}{x-1}$

a) Domain: $\{x \mid x \neq 1\}$

b) Vertical Asymptote(s): $x = 1$

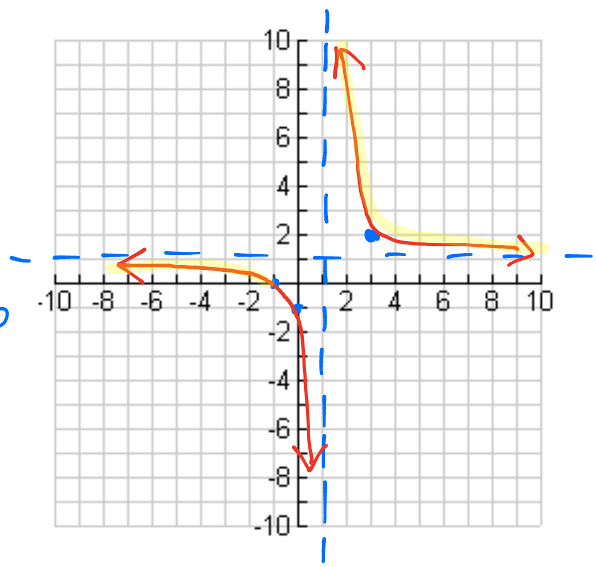
c) x-intercept(s): $(-1, 0)$ $x+1=0$, $x=-1$

d) y-intercepts(s): $(0, -1)$ $\frac{0+1}{0-1}=\frac{1}{-1}$

e) End Behavior Asymptote: $y = 1$ $y=\frac{x}{x}=1$

$x-1 \overline{)\,x+1}$ quotient $1 + \frac{2}{x-1}$, $-(x+1)$, remainder 2; $\frac{2}{x-1} \to 0$

f) Graph the function.



Test: $x = 3$

$\frac{3+1}{3-1} \cdot \frac{4}{2} = 2$

g) Limits of the ends and near each vertical asymptote:

h) Looking only at $\dfrac{x+1}{x-1} > 0$, find the solutions for x.

$\lim_{x \to -\infty} f(x) = 1$ $\lim_{x \to +\infty} f(x) = 1$

$\lim_{x \to 1^-} f(x) = -\infty$ $\lim_{x \to 1^+} f(x) = \infty$

Solution: $(-\infty, -1) \cup (1, \infty)$

6. $f(x)=\dfrac{(x+5)^2}{x^2-4}$ $\dfrac{(x+5)(x+5)}{(x+2)(x-2)}$ $\dfrac{x^2+10x+25}{x^2-4}$

a) Domain: $\{x \mid x \neq \pm 2\}$

b) Vertical Asymptote(s): $x=\pm 2$

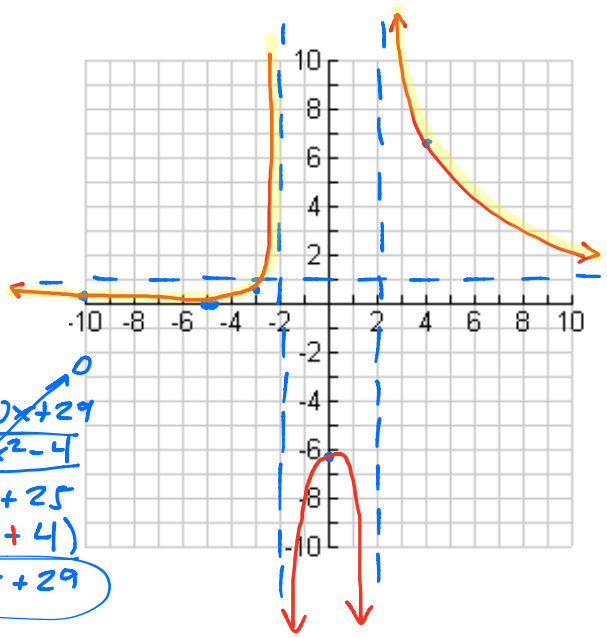
c) x-intercept(s): $(-5,0)$ mult. of 2 $x+5=0$, $x=-5$

d) y-intercepts(s): $\left(0,-\frac{25}{4}\right)$ $\frac{(0+5)^2}{0^2-4}=\frac{25}{-4}$

e) End Behavior Asymptote: $y=\frac{x^2}{x^2}=1$ $y=1$

$$x^2+0x-4 \;\overline{)\; x^2+10x+25}$$
$$1 + \frac{10x+29}{x^2-4}$$
$$-(x^2+0x+4)$$
$$10x+29$$

f) Graph the function.



g) Limits of the ends and near each vertical asymptote:

$x=-10$ $\frac{(-5)^2}{100-4}$ $\frac{25}{96}$

$x=-3$ $\frac{(2)^2}{5}$ $\frac{4}{5}$

$x=4$ $\frac{(9)^2}{12}$ $\frac{81}{12}$

h) Looking only at $\dfrac{(x+5)^2}{x^2-4} \geq 0$, find the solutions for x.

$\lim_{x\to-\infty} f(x)=1$ $\lim_{x\to+\infty} f(x)=1$

$\lim_{x\to-2^-} f(x)=-\infty$ $\lim_{x\to-2^+} f(x)=-\infty$

$\lim_{x\to2^-} f(x)=-\infty$ $\lim_{x\to2^+} f(x)=+\infty$

Solution: $(-\infty,-2)\cup(2,\infty)$