1

8.3 - 8.5 Test Review

$\sin^2\theta + \cos^2\theta = 1$
$1 + \cot^2\theta = \csc^2\theta$
$\tan^2\theta + 1 = \sec^2\theta$

$\boxed{8.3}$

1. $(\csc\theta - 1)(\csc\theta + 1) = \cot^2\theta$

$$\csc^2\theta + \cancel{\csc\theta} - \cancel{\csc\theta} - 1 = \cot^2\theta$$
$$\underbrace{\csc^2\theta - 1}_{\cot^2\theta} = \cot^2\theta$$
$$\cot^2\theta = \cot^2\theta \; //$$

2. $(1 - \cos^2\theta)(1 + \cot^2\theta) = 1$

$$(\sin^2\theta)(\csc^2\theta) = 1$$
$$\frac{\cancel{\sin^2\theta}}{1} \cdot \frac{1}{\cancel{\sin^2\theta}} = 1$$
$$1 = 1 \; //$$

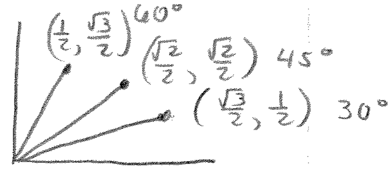
3. $\tan^2\theta \cos^2\theta + \cot^2\theta \sin^2\theta = 1$

$$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1} + \frac{\cos^2\theta}{\cancel{\sin^2\theta}} \cdot \frac{\cancel{\sin^2\theta}}{1} = 1$$
$$\underbrace{\sin^2\theta + \cos^2\theta}_{1} = 1$$
$$= 1 \; //$$

4. $\dfrac{\sec\theta}{\csc\theta} + \dfrac{\sin\theta}{\cos\theta} = 2\tan\theta$

$$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} + \tan\theta = 2\tan\theta$$
$$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} + \tan\theta = 2\tan\theta$$
$$\frac{\sin\theta}{\cos\theta} + \tan\theta = 2\tan\theta$$
$$\tan\theta + \tan\theta = 2\tan\theta$$
$$2\tan\theta = 2\tan\theta \; //$$

②



$\boxed{8.4}$

① $\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)$

$= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \dfrac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$ $\qquad \tan\frac{\pi}{6} = \cancel{\frac{1}{2}} \cdot \cancel{\frac{2}{\sqrt{3}}} = \frac{\sqrt{3}}{3}$

$= \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)}$

$= \dfrac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}}$

$= \dfrac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} \cdot \dfrac{\frac{3}{3+\sqrt{3}}}{\frac{3}{3+\sqrt{3}}}$

$= \dfrac{3-\sqrt{3}}{\cancel{3}} \cdot \dfrac{\cancel{3}}{3+\sqrt{3}}$

$= \dfrac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \dfrac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \dfrac{9 - 6\sqrt{3} + 3}{9 - 3} = \dfrac{12 - 6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$

② $\sin 165° = \sin(135° + 30°)$

$= \sin 135° \cos 30° + \cos 135° \sin 30°$

$= \left(\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) + \left(\frac{-\sqrt{2}}{2} \cdot \frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

$= \boxed{\dfrac{\sqrt{6} - \sqrt{2}}{4}}$

③ $\cos\frac{17\pi}{12} = \cos\left(\frac{15\pi}{12} + \frac{2\pi}{12}\right)$

$= \cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{5\pi}{4}\cos\frac{\pi}{6} - \sin\frac{5\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{2}}{2} \cdot \frac{1}{2}\right)$

$= \frac{-\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\dfrac{-\sqrt{6} + \sqrt{2}}{4}}$

3

8.4

4) $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$

$= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$= \dfrac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}$ $\qquad \tan\frac{\pi}{6} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$= \dfrac{\frac{\sqrt{3}}{3} + 1}{1 - \left(\frac{\sqrt{3}}{3} \cdot 1\right)}$

$= \dfrac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$

$= \dfrac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}}$

$= \dfrac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}}$

$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$= \dfrac{9 + 6\sqrt{3} + 3}{9 - 3} = \dfrac{\cancel{12} + \cancel{6}\sqrt{3}}{\cancel{6}} = \boxed{2 + \sqrt{3}}$

5) $\sin(20° - 80°)$

$= \sin(-60°)$

$= -\sin(60°)$

$= -\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{3}}{2}$

6) $\cos\left(\frac{5\pi}{12} + \frac{7\pi}{12}\right)$

$= \cos\left(\frac{12\pi}{12}\right)$

$= \cos\pi$

$= -1$

7) $\tan(40° - 10°)$

$= \tan(30°)$

$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{3}$

8) $\tan\left(\frac{7\pi}{18} - \frac{\pi}{18}\right)$

$= \tan\left(\frac{6\pi}{18}\right)$

$= \tan\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1}$

$= \sqrt{3}$

(4)

$\boxed{8.4}$

① $\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\cos\frac{\pi}{2}\cos\theta - \sin\frac{\pi}{2}\sin\theta = -\sin\theta$

$\cancel{(0)(\cos\theta)} - (1)(\sin\theta) = -\sin\theta$

$0 - \sin\theta = -\sin\theta$

$-\sin\theta = -\sin\theta$ //

② $\tan(2\pi-\theta) = -\tan\theta$

$$\frac{\tan 2\pi - \tan\theta}{1+(\tan 2\pi)(\tan\theta)} = -\tan\theta$$

$$\frac{0 - \tan\theta}{1+\cancel{(0)(\tan\theta)}} = -\tan\theta$$

$$\frac{-\tan\theta}{1} = -\tan\theta$$

$-\tan\theta = -\tan\theta$ //

Changed → ③ $\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} = \tan\alpha + \tan\beta$

$$\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} = \tan\alpha\tan\beta$$

$$\frac{(\sin\alpha)\cancel{(\cos\beta)}}{(\cos\alpha)\cancel{(\cos\beta)}} + \frac{\cancel{(\cos\alpha)}(\sin\beta)}{\cancel{(\cos\alpha)}(\cos\beta)} = \tan\alpha\tan\beta$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \tan\alpha\tan\beta$$

$\tan\alpha + \tan\beta = \tan\alpha\tan\beta$ //

④ $\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$

$\sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta} = 2\sin\alpha\cos\beta$

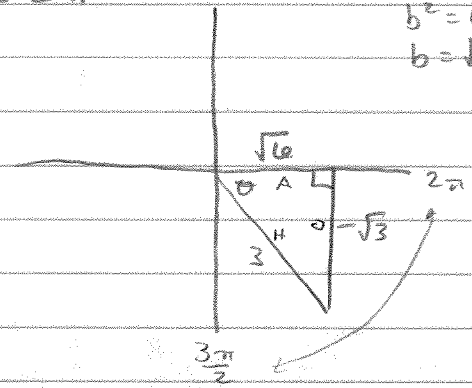
$2\sin\alpha\cos\beta = 2\sin\alpha\cos\beta$ //

(5)

$\boxed{8.5}$

$$(-\sqrt{3})^2 + b^2 = (3)^2$$
$$3 + b^2 = 9$$
$$b^2 = 6$$
$$b = \sqrt{6}$$

(1) $\sin\theta = -\frac{\sqrt{3}}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$



a) $\sin(2\theta)$

$$= 2\sin\theta\cos\theta$$
$$= 2\left(-\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{6}}{3}\right)$$
$$= \frac{-2\sqrt{18}}{9} = \frac{-6\sqrt{2}}{9}$$
$$= \frac{-2\sqrt{2}}{3}$$

b) $\cos(2\theta)$

$$= \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{\sqrt{6}}{3}\right)^2 - \left(-\frac{\sqrt{3}}{3}\right)^2$$
$$= \frac{6}{9} - \frac{3}{9} = \frac{3}{9} = \frac{1}{3}$$

c) $\sin\left(\frac{\theta}{2}\right)$ The half angle will be between $\frac{3\pi}{4}$ and π. Q II

$$= +\sqrt{\frac{1-\cos\theta}{2}}$$
$$= \sqrt{\frac{1-\left(\frac{\sqrt{6}}{3}\right)}{2}}$$
$$= \sqrt{\frac{\frac{3-\sqrt{6}}{3}}{2}}$$
$$= \sqrt{\frac{3-\sqrt{6}}{3}\cdot\frac{1}{2}}$$
$$= \sqrt{\frac{3-\sqrt{6}}{6}}$$

d) $\cos\left(\frac{\theta}{2}\right)$

$$= -\sqrt{\frac{1+\cos\theta}{2}}$$
$$= -\sqrt{\frac{1+\left(\frac{\sqrt{6}}{3}\right)}{2}}$$
$$= -\sqrt{\frac{\frac{3+\sqrt{6}}{3}}{2}}$$
$$= -\sqrt{\frac{3+\sqrt{6}}{3}\cdot\frac{1}{2}}$$
$$= -\sqrt{\frac{3+\sqrt{6}}{6}}$$