Pre Calculus
5.1 Notes – Polynomials

Name _____
Block ____

Key Words:
Polynomial
Degree
Leading Coefficient
Constant
Multiplicity
Turning Point
End behavior / Limits

A **POLYNOMIAL** is a function of the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + \ldots a_1 x^1 + a_0$$

constant

where n represents a non-negative integer

degree

Example: $f(x) = 3x^5 + 2x^3 - 4x^2 - x + 9$ constant

Degree - The largest exponent. 5

Power Function: f(x) = $3x^5$

Leading Coefficient - The number in front of the variable of largest exponent. 3

Constant - The unchanging number. A number without a variable. 9

Your turn…

1. Determine which of the following are polynomial functions. If yes, state the degree and leading coefficient. If not, explain why it is not a polynomial function.

a) $f(x) = x^4 - 8x^{-2} + 9x - 12$ Yes or (No)

Degree_____ Leading Coefficient _____ or Explain -2 is a neg. integer

b) $g(x) = 5x^{12} + 10x^8 - 1$ (Yes) No

Degree 12 Leading Coefficient 5 or Explain

c) $h(x) = -18x^2 - 4x^3 + 12 - 5x^6$ Yes No

Degree 6 Leading Coefficient -5 or Explain

d) $k(x) = 13\sqrt{x}$ $13x^{\frac{1}{2}}$ Yes (No)

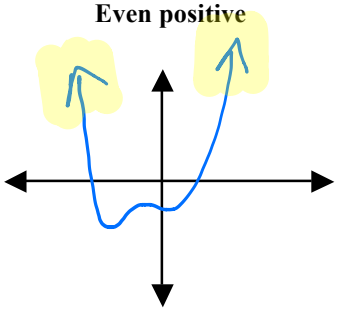
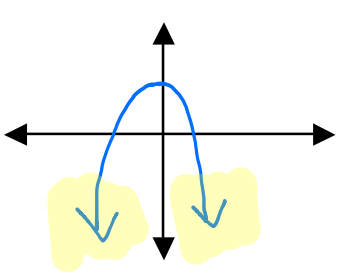
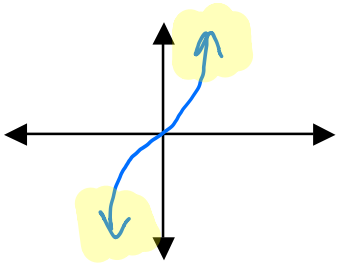
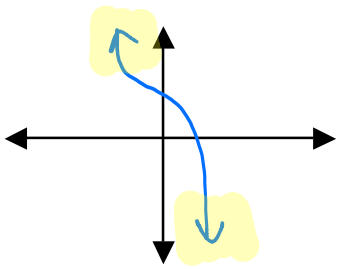
Degree_____ Leading Coefficient _____ or Explain

$\frac{1}{2}$ is not an integer

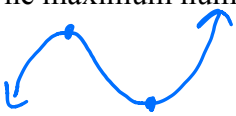
ARROWS

End behavior: The action/direction of the **"end"** of the graph of a function. a.k.a. **LIMITS**

$3x^6+3x^3-2$ $-5x^4+x^3-12x^3+4$ $4x^3-x^2+2x$ $-5x^5+x^2-x+3$

Even positive	**Even negative**	**Odd positive**	**Odd negative**
			
$\lim_{x\to\infty} f(x) = +\infty$ Right Up	$\lim_{x\to\infty} f(x) = -\infty$ Right	$\lim_{x\to\infty} f(x) = +\infty$ R	$\lim_{x\to\infty} f(x) = -\infty$
$\lim_{x\to-\infty} f(x) = +\infty$ Left Up	$\lim_{x\to-\infty} f(x) = -\infty$ Left	$\lim_{x\to-\infty} f(x) = -\infty$ L	$\lim_{x\to-\infty} f(x) = +\infty$

Turning point: Local maxima or local minima. The maximum number of turning points is (n-1), one less than the degree



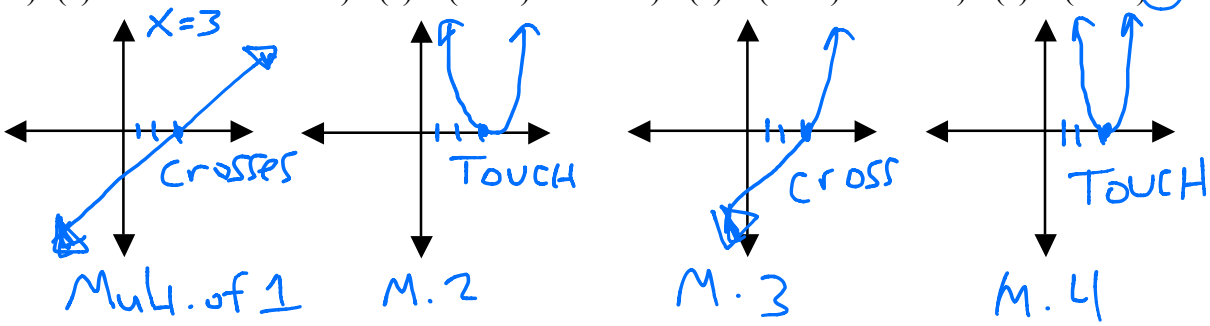
Multiplicity: The number of times a zero occurs.

x^2 1 T.P.

Your turn…

Sketch a graph of:

a) $f(x) = x - 3$ b) $f(x) = (x-3)^2$ c) $f(x) = (x-3)^3$ d) $f(x) = (x-3)^4$

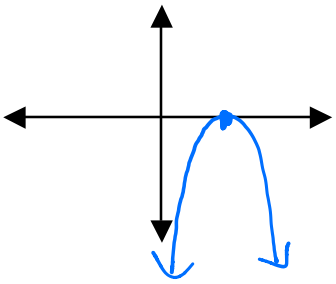


Mult. of 1 M.2 M.3 M.4

so…we can generalize…

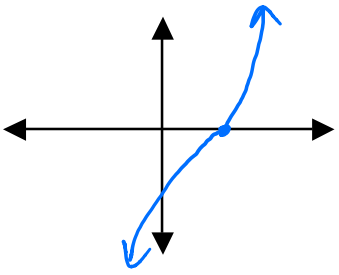
EVEN MULTIPLICITY

"Touch" the x-axis and go back down



ODD MULTIPLICITY

"Cross" the x-axis



Let's GRAPH!!!

1. Find the following for: $k(x) = -x^3 - x^2 + 12x$

a) Determine the zeros and their multiplicity and whether they cross or touch the x-axis.

$0 = -x(x^2 + x - 12)$

$0 = -x(x+4)(x-3)$

$-x = 0$ → $x = 0$, M.1, C

$x + 4 = 0$ → $x = -4$, M.1, C

$x - 3 = 0$ → $x = 3$, M.1, C

b) Determine the degree.

3

c) Determine the maximum possible number of turning points.

2

d) Find the *y*-intercept.

$(0, 0)$

$-(0)^3 - (0)^2 + 12(0)$

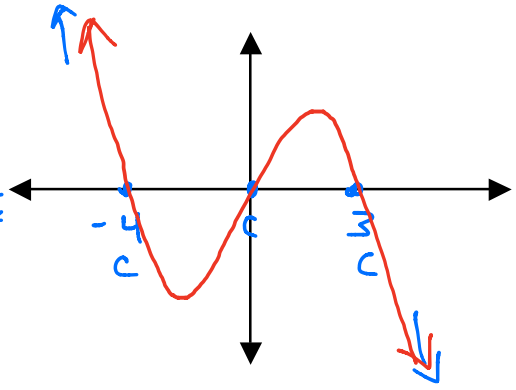
e) Determine the end behavior of *f*(x).

* **Power function**: f(x) = $-x^3$ (ODD, NEGATIVE)

$\lim_{x \to \infty} f(x) = -\infty$ Right

$\lim_{x \to -\infty} f(x) = +\infty$ Left

f) Sketch the graph of the function.



2. Find the following for: $h(x) = (x-3)^2(x+2)$

a) Determine the zeros and their multiplicity and whether they cross or touch the x-axis.

$0 = x - 3$ → $x = 3$, M.2, TOUCH

$0 = x + 2$ → $x = -2$, M.1, CROSS

b) Determine the degree.

$x^2 \cdot x^1 = x^3$

3

c) Determine the maximum possible number of turning points.

2

d) Find the *y*-intercept.

$(0, 18)$

$(0-3)^2(0+2)$

$(-3)^2(2)$ $\quad 9 \cdot 2 = 18$

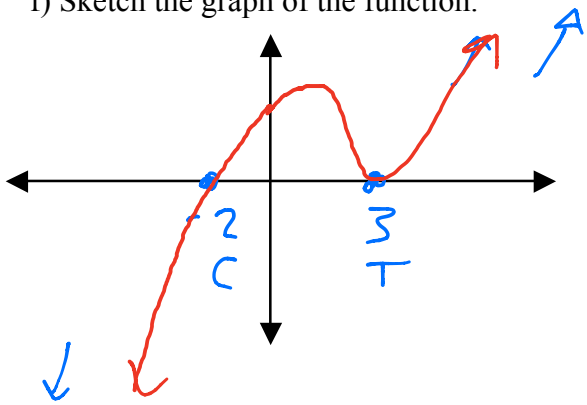
e) Determine the end behavior of *f*(x).

Power function: f(x) = x^3 (ODD, POS.)

$\lim_{x \to \infty} f(x) = +\infty$ R

$\lim_{x \to -\infty} f(x) = -\infty$ L

f) Sketch the graph of the function.



Your turn…

3. Find the following for: $h(x) = -x^2(x^2 - 4)(x - 5)$

a) Determine the zeros and their multiplicity and whether they cross or touch the x-axis.

$0 = -x^2$ $0 = x^2 - 4$ $0 = x - 5$

$x = 0$ $4 = x^2$ $x = 5$

M.2 $x = 2$ $x = -2$ M.1

T M.1 M.1 C

C C

b) Determine the degree.

5

$x^2 \cdot x^2 \cdot x^1$

c) Determine the maximum possible number of turning points.

4

d) Find the *y*-intercept.

(0, 0)

$-(0)^2(0^2 - 4)(0 - 5) = 0$

e) Determine the end behavior of *f*(x).

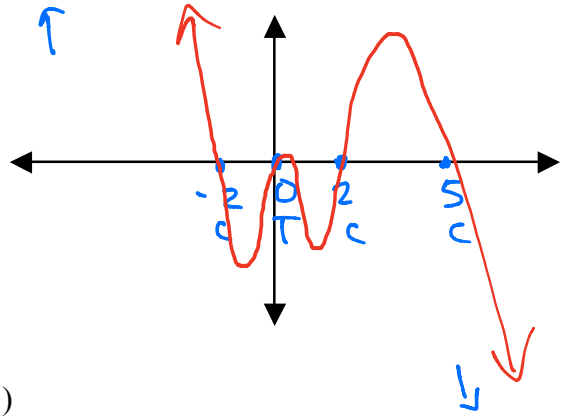
Power function: f(x) = $-x^5$ ODD

$\lim_{x \to \infty} f(x) = -\infty$ R

$\lim_{x \to -\infty} f(x) = +\infty$ L

NEG.

f) Sketch the graph of the function.



4. Find the following for: $g(x) = (x - 1)^2(x - 3)(x + 1)$

a) Determine the zeros and their multiplicity and whether they cross or touch the x-axis.

$0 = x - 1$ $0 = x - 3$ $0 = x + 1$

$x = 1$ $x = 3$ $x = -1$

M.2 M.1 M.1

T C C

b) Determine the degree.

4

c) Determine the maximum possible number of turning points.

3

d) Find the *y*-intercept.

(0, -3)

$(0-1)^2(0-3)(0+1) = (1)(-3)(1)$

e) Determine the end behavior of *f*(x).

Power function: f(x) = x^4 EVEN

$\lim_{x \to \infty} f(x) = +\infty$ POSITIVE

$\lim_{x \to -\infty} f(x) = +\infty$

f) Sketch the graph of the function.