# Lincoln Public Schools

# Secondary (6-12) Mathematics Curriculum Guide

**Revised June 2014**

## Table of Contents

**Goals for Students**

The Lincoln Public Schools Mathematics Program has four broad goals for students:

1. Students will acquire mathematical skills, including the ability to perform routine computations. At the secondary level, this includes traditional, but routine, symbolic manipulation.

2. Students will develop an understanding of fundamental mathematical concepts.

3. Students will become mathematical problem solvers.

4. Students will learn to value mathematics and the quantitative nature of our world.

**Curriculum**

*Standards Instructional Objectives*

The secondary math program is based on a set of specific standards and course objectives. The district math standards are the Nebraska State Math Standards and specific course objectives/syllabi have been designed to ensure students have the opportunity to acquire the knowledge and skills outlined in the Nebraska State Math Standards. The district math objectives and the syllabus for each course are available on DocuShare. The objective cards/syllabi not only outline the specific course objectives, but in addition they contain pacing information, student assessment information, Nebraska State Standards and NeSA-M correlations, and furnish useful information to students, teachers, administrators, and parents. For certain courses, the district has generated additional supplemental materials and resources. All supplemental materials and resources (course assessments) are available on DocuShare.

# Middle Level Mathematics Program

***Typical Middle Level Course Sequences***

**Option 1** (mainstream with or without math intervention):

| Grade 6 → | Grade 7 → | Grade 8 |
|---|---|---|
| Course Title: Math 6 | Course Title: Math 7 | Course Title: Math 8 |
| Text: Holt Course 1, ©2004 | Text: Holt Course 2, ©2004 | Text: McDougal Littell Course 3, ©2004 |

**Option 2**:

| Grade 6 → | Grade 7 → | Grade 8 |
|---|---|---|
| Course Title: Math 6 | Course Title: Prealgebra | Course Title: Algebra |
| Text: Holt Course 1, ©2004 | Text: McDougal Littell Course 3, ©2004 | Text: *Algebra 1, Prentice Hall © 2009*. |

**Option 3** (high achiever option – student skips seventh grade math):

| Grade 6 → | Grade 7 → | Grade 8 |
|---|---|---|
| Course Title: Math 6 | Course Title: Pre-Algebra | Course Title: Algebra |
| Text: Holt Course 1, ©2004 | Text: Holt Course 2, ©2004 | Text: Algebra 1, *Prentice Hall © 2009* |

**Option 4** (Differentiated Program):

| Grade 6 → | Grade 7 → | Grade 8 |
|---|---|---|
| Course Title: Diff Math 6 | Course Title: Prealgebra D | Course Title: Algebra D |
| Text: McDougal Littell Course 2, ©2004 | Text: McDougal Littell Prealgebra, ©2005 | Text: Algebra 1, *Prentice Hall © 2009* |

# Middle Level Math Placement Criteria

<u>**General Guidelines**</u>

- Place students in the highest possible math class ***subject to the criteria below***.

- **A student's skill is the most important factor.** Work habits are not a defining consideration. This means completion of homework is not a factor unless this contributed to the lack of student understanding which would be reflected in a student's test scores and ultimately their academic progress grade.

- **No one score or criterion should be used. The single most important determinant is teacher recommendation.**

<u>**Sixth to Seventh Placement**</u>

***Option 1: Prealgebra D (Diff course for 7th graders)***

Students placed in this course should include the following:

- VERY Successful Math 6D students – students earning B or above
- BORDERLINE successful Math 6D students – earning C/C+
  - following a discussion with 7th grade teacher and liaison
  - following a parent/contact explaining potential challenge for the student based on borderline performance

***Option 2: Prealgebra (High Achiever course for 7th grade)***

Students placed in this course should include the following:

- Students who struggled (earned below a C) in Math 6D due to a true lack of understanding. Parent contact made to explain the need for non-differentiated placement, but also the promise of algebra placement in 8th grade upon successful completion of this course.
- BORDERLINE successful Math 6D students – earning C/C+
  - following discussion with 7th grade teacher and liaison
  - following parent/contact to explain the decision and request approval/support (particularly if student is identified "Gifted")
- Highly successful Math 6 students – earning A/B+ and "bored" or "ready" for greater challenge
  - The target student for this course is a very high achiever in sixth grade math and who has not previously been identified for the gifted program.
  - Successful performance will lead to recommendation for algebra in 8th grade

***Option 3: Math 7***
Students placed in this course are Math 6 students who do not fit under option 2.

- Students needing math intervention would be required to enroll in this regular math course.

**Seventh to Eighth**

***Option 1: Algebra D (Diff course for 8th graders)***
Students placed in this course should include the following:

- All clearly successful Prealgebra D students – Grade of B or above
- BORDERLINE & NOT Identified GIFTED Prealgebra D students – Grade of C/C+
    - following discussion with 7th grade teacher and liaison
    - following parent/contact explaining potential challenge for the student based on borderline performance
- BORDERLINE or below and GIFTED Prealgebra D students – Grade of C or below
    - following discussion with 7th grade teacher and liaison
    - following parent/contact explaining potential challenge for the student based on performance

***Option 2: Algebra***
Students placed in this course should include the following:

- All clearly successful Prealgebra students – Grade of B or above
- BORDERLINE successful Prealgebra students – earning C/C+
    - following a discussion with 7th grade teacher and liaison
- Students who struggled (earned below a C) in Prealgebra D due to a true lack of understanding.
    - Parent contact made to explain the need for non-differentiated placement (particularly if student has "gifted" label), but still on track to meet high school algebra graduation requirement

***Option 3: Math 8 (Prealgebra Course)***
Students placed in this course are 7th grade Prealgebra students who do not fit under option 2 and all Math 7 students

- Students needing math intervention would be required to enroll in this regular math course.

**Mid-year Placement Adjustments**

- ***6th grade:*** **When in doubt** (a borderline or "on the bubble" type student) sixth grade teachers should meet with 7th grade teachers and/or the math liaison to discuss the recommendation.
    - NOTE: Extremely successful Math 6 students earning an A and "bored" or "ready" for greater challenge should be considered for placement in the Math 6D course by 2nd semester

- ***7th grade:*** **When in doubt** (a borderline or "on the bubble" type student) seventh grade teachers should meet with 8th grade teachers and/or the math liaison to discuss the recommendation.
    - NOTE 1: Extremely successful Math 7 students earning an A/B+ and "bored" or "ready" for greater challenge should be considered for a move to Prealgebra by 2nd semester, in order to potentially prepare them for algebra placement in 8th grade.
    - NOTE 2: Extremely successful Prealgebra students earning an A and "bored" or "ready" for greater challenge should be considered placement in the Prealgebra D course by 2nd semester.

## Curriculum-Based Middle Level Math Interventions

### Purpose

Research indicates that middle level students who struggle in mathematics can be successful in on grade level course work provided they receive additional instructional time and support (Burris, Heubert, & Levin, 2006). To be effective, this additional instruction and support must be in addition to and integrated with the regular classroom (Balfanz, Mac Iver, & Byrnes, 2006). Therefore, the purpose of middle level math intervention is to allow all students, with the exception of functional/life skills students, access to the regular curriculum with the support they need to be successful. Math Intervention is an **extension** of the regular grade level course that provides students who need it additional focused instruction and support at the needed level of intensity. No student should be placed in math intervention if she/he is not also enrolled in the corresponding grade level math course. That is, no student should be enrolled in math intervention as his/her sole math course.

*Middle Level Math Intervention Goals*

- To increase student self-efficacy in mathematics.
- To support students in mastering grade level math standards.
- To re-teach critical concepts and skills based on DCA results to prepare for NeSA-M.
- To fill pre-requisite grade level skill gaps.

### Tier1: Classroom Re-teaching

Classroom re-teaching and re-learning takes place daily, within the grade level course, based on formative assessments (homework and or quizzes). Often student needs are addressed through warm-up problems or other instructional activities. Consistent spiral review and effective use of NeSA-M review materials is necessary to support student retention.

## Tier1+: Fluid Math Support

*Structure*

The structure of this course is truly "fluid" with students entering and exiting as needs dictate. Students rotate out of an academic connections course and into the fluid math support class to receive additional instruction that focuses on a particular math concept the student has not yet mastered. A fluid math cycle may last one to two weeks. For maximum effectiveness, course size should be 15 students or less. This fluid structure requires quality communication, tight logistics, and a strong teamwork approach among building staff in order to be successful. Fluid Math is not a distinct course, i.e. it is not identified separately on a student progress report. Student progress within a fluid math cycle will be reflected through his/her regular math course grade. Progress/Report card comments should be used to acknowledge the connection between a student's time spent in a fluid math cycle and improved understanding of grade-level math content.

*Placement*

The concepts considered for re-teaching and re-learning in a fluid math cycle are identified during PLC work and based on formative and summative assessment data. Therefore, student performance on assessments determines his/her placement in a fluid math cycle. Teachers typically recommend students with borderline below proficient scores for this placement because it is believed that these "on the bubble" students can most benefit from a short time-period of intensive re-teaching and re-learning in order to reach proficiency.

It is important to communicate with parents at the start of the school year the purpose of Fluid Math Intervention. A letter explaining the student benefits and placement logistics for fluid math intervention is found in this guide. The building principal is responsible for sending this communication to families.

*Components of Fluid Math Support Lessons*

Math Intervention lessons should be designed so that students have the opportunity to "experience" mathematics through a variety of learning tasks. Learning tasks need to vary, rather than simply re-stating initial math instruction, but slower and louder. Although the goal of Intervention is not necessarily to make math "fun," by varying the learning tasks and making the instruction engaging the Intervention teacher has the opportunity to reach more students and build success. The research indicates students "enjoy" those subjects in which they experience success, the ultimate goal of Intervention. Components of this learning experience include:

- Direct instruction – **Math Intervention is not a homework/study hall period.**
- Re-teaching of regular course content with emphasis on results of District Common Assessments for Math (DCA-M = Benchmark).
- Hands-on learning tasks with extensive modeling.
- Additional guided practice.
- Emphasis on reading, understanding, and representing problems – the problem solving process.

- Classroom discussion and "math talk" – there should be a great deal of teacher-student interaction during class.
- Emphasis on teaching math vocabulary – word walls.
- Spiral review (use of NeSA-M review materials)
- Addressing pre-requisite skill gaps
- Use of appropriate Technology (geogebra, virtual manipulatives) to enhance the learning experience.

**Tier2: Math 180 (Title1 Schools)**

*Structure*

The Math 180 program provides additional math instruction and support to reach students who are performing **significantly below grade level**. Direct instruction is coupled with interactive technology that adapts to provide practice meeting individual student needs. Another integral component of the Math180 program is its emphasis on fostering positive student attitudes toward learning mathematics (development of a growth mindset). This program aims to close the achievement gap and allow on grade level content to be more accessible for students who are generally 2 or more grade levels behind. The Math 180 course is an **additional** period of daily math instruction, taking the place of an academic connections course. No student should be enrolled in Math 180 as his/her sole math course. For maximum effectiveness, course size should be 15 students or less. The technology component of this course requires that each student have daily access to computers (and headphones), and that the teacher's computer be linked to a projector or smartboard system to deliver instruction.

Students enrolled in the Math 180 course are to be **graded** on an **"individual basis"** earning progress marks of *Commendable (C), Satisfactory (S)* or *Needs Improvement (N)* which are reflective of the student's work in the course.

*Placement*

Placement for Math180 is based on teacher recommendation. For students who will be in 7th or 8th grade, placement recommendations should be made in May for the next school year. For students who will be in 6th grade, placement in the course should also be based on teacher recommendation, but take place after the first summative assessment in Math 6.

Teachers should consider the following student characteristics when making recommendations for Math 180:

- Students with **significant** skill gaps, performing below grade level.
- Students who consistently fail course assessments.
    - Middle School gradebook shows Big Idea Scores below 50%
    - Elementary Progress Report shows scores of 1's and 2's, particularly in Number Sense.
- Students for whom Tier1+ intervention is not sufficient to support them in meeting

grade level standards.
- Students who perform Below the Standards on NeSA-M (scale score < 84)

Once a student is placed in the Math 180 course, a baseline assessment is administered and can be used to verify appropriate placement.

**Progress/Final academic mark for Math Intervention**

Because Math Intervention is an extension of the regular math course and is intended to support student mastery of the regular objectives. Math Intervention is to be graded on **an "individual" basis** and is given either *satisfactory (S)* or *needs improvement (N)* under the progress/final grade column. Commendable (C) is not an option as this sometimes sends parents a false message concerning the student's academic achievement in mathematics. Generally, students of this nature do not need to be in Math Intervention.

S The *Satisfactory* student consistently uses his/her time in Math Intervention to improve his/her understanding of grade level objectives and this is reflected in the student's regular course success. This implies that communication has taken place between classroom and Intervention teacher if they are not the same person.

N The *Needs Improvement* student does not consistently use his/her time in Math Intervention to improve his/her understanding of grade level objectives and this is reflected in the student's regular course success. This implies that communication has taken place between classroom and Intervention teacher if they are not the same person.

**Sample MIDDLE SCHOOL REPORT CARD**

March 19, 2007
To the Parents/Guardian of:
PATTY HAMMOND
6720 S 43RD ST
LINCOLN NE 68516

| | | | |
|---|---|---|---|
| Student Name: | PATTY HAMMOND | Student No.: | 666666 |
| School: | SCOTT | Grade: | 08 |
| Counselor: | N TEGLER | Year: | 2006 - 07 SEM 1 |

| Course | Instructor | Progress Grades | Final Grades | Basis | Work Study Habits | Social Behavior Skills |
|---|---|---|---|---|---|---|
| | | | | | | |
| **Math Intervention**<br>**Participates constructively in class** | **M. Larson** | **Mark should be S or N Use descriptors below. NO ACADEMIC GRADE SHOULD EVER BE GIVEN HERE.** | | **IND** | **C, S, N Use the district descriptors.** | **C, S, N Use the district descriptors.** |

**Total Absences:** 2.25 **Total Tardies:** 1

**Meaning of Marks**
A Superior C D Passing W Withdrew
B+ C+ F Failing S Satisfactory COM Commendable
B D+ INC Incomplete U Unsatisfactory N Needs Improvement

**Work Study Habits** | **Social Behavioral Skills** | **Basis**
COM – Commendable | COM – Commendable | CSO - Curriculum/Standards/Objectives
**S – Satisfactory** | S – Satisfactory | **IND - Individual**
**N - Needs Improvement** | N – Needs Improvement

**Sample Math Intervention Parent Letter**

Date, 20XX

Dear Parent/Guardian:

At _________ Middle School our primary concern is for your student's academic success. We want to do all we can to support students as they progress through middle school and help ensure that students will be able to meet high school graduation requirements. In order to support students, we offer interventions in reading and math for students who may benefit academically from them.

We offer a math support called "Fluid Math Intervention." Each level of math has a number of "Big Idea" concepts taught each semester. Students who do not initially demonstrate mastery of these concepts on chapter tests will be given an opportunity to re-learn the concepts during Fluid Math Intervention. Fluid Math Intervention is in addition to your son's/daughter's regular math course and is a temporary assignment made in place of one of the student's Academic Connections classes. Each placement is for one week where your student will have an opportunity to re-learn critical math concepts. After the weeklong re-teaching session your student will return to his/her Academic Connection class. The student will not be responsible for the work he/she missed in his/her Academic Connection class. The pattern will repeat as necessary for each of the major concepts taught in math during the year.

The purpose of this math intervention is to provide your student with additional direct math instructional time and to address concepts and skills that were not mastered in order to help keep him/her on grade level. Longitudinal data on high school students' success indicate that students who are enrolled in the appropriate mathematics courses greatly enhance their probability of meeting the mathematics graduation requirement in a timely manner.

Our professional recommendation to place your student in Fluid Math Intervention throughout the year is made with your student's current and long term academic interests in mind.

Sincerely,

Principal
_______ Middle School

**Math Intervention Waiver Form**

My signature below acknowledges that I DO NOT wish to have my student enrolled in mathematics intervention and understand the following:

1. Staff at _________ recommends that in order to maximize my student's probability of success in mathematics in middle school and in high school that he/she be enrolled in mathematics intervention.
2. I will not hold the Lincoln Public School District liable for my student's need for additional mathematics instruction during high school, nor for any additional time my student may require to complete his/her high school graduation requirements.

Signed: ___________________________________________________ Date: ________
(Parent/Guardian signature)

Signed: ___________________________________________________ Date: ________
(Parent/Guardian signature)

Signed: ___________________________________________________ Date: ________
(Middle School Principal, Lincoln Public Schools)

Note: Waiver must be completed annually.

Copies: Parent
Student's Cumulative File

**Middle Level Course Descriptions**

*Math 6*

This first secondary mathematics course introduces students to the core mathematics strands: computation, measurement, geometry, algebra, data analysis, and probability. One of the major themes of this course is problem solving with decimals and fractions.

Text: Holt Course 1, ©2004.

*Math 6D*

This first secondary mathematics course introduces students to the core mathematics strands: computation, measurement, geometry, algebra, data analysis, and probability. The differentiated course covers additional topics in greater depth than Math 6.

Text: McDougal Littell Course 2, ©2004

*Math 7*

In this second course students continue their study of the core mathematics strands. The major emphasis of this course is on proportional reasoning, integers, geometry, and algebra readiness.

Text: Holt Course 2, ©2004.

*Prealgebra*

*Prealgebra* is an accelerated course for seventh grade students. Students in this course skip traditional seventh grade math and instead focus on preparation for algebra in grade 8.

Text: McDougal Littell Course 3, ©2004.

*Prealgebra D*

*Prealgebra D* is an accelerated and differentiated course. Students in this course skip traditional seventh grade math and instead focus on a rigorous preparation for algebra in grade 8.

Text: McDougal Littell Prealgebra, ©2005.

*Algebra and Algebra D**

Algebra is the first course in the traditional college preparatory sequence. Course topics include equation solving, linear sentences, linear inequalities, lines, slope, graphing, exponents and powers, polynomials, systems of equations, quadratic equations, functions, and statistics. This course is available in middle school in both a regular and differentiated version.

Text Algebra: *Algebra 1,* ©2009 Prentice Hall Mathematics.
Text Algebra D: *Algebra 1,* ©2009 Prentice Hall Mathematics.

*It is critical to note that algebra in grade 8 counts towards high school graduation requirements. Specifically, students earn high school credit for the course and a student's grade in the course is a factor in a student's high school GPA and class rank.

## High School Mathematics Program

### Graduation Requirement

The high school graduation requirement is 30 hours of mathematics, including course work in algebra and geometry. The 30 hours of mathematics credit must come from courses beginning with Algebra/Algebra Block (special education students will graduate according to requirements outlined in their IEP). This does not mean that a student has to pass each semester of algebra and geometry to graduate, only that a student's 30 hours of mathematics must include, at a minimum, 5 hours of algebra credit and 5 hours of geometry credit.

### Typical High School Course Sequences*

| Grade 9 | Grade 10 | Grade 11 | Grade 12 |
|---|---|---|---|
| Option 1: | | | |
| Course Title:<br>**Algebra Block** | Course Title:<br>**Geometry Plus 1** | Course Title:<br>**Geometry Plus 2/ Elements of Advanced Alg** | Course Title:<br>**Advanced Algebra** |
| Text: Algebra 1<br>Prentice Hall © 2009 | Text: Geometry<br>Holt McDougal, ©2012 | Text: Algebra 2<br>Prentice Hall © 2009<br>Geometry<br>Holt McDougal, ©2012 | Text: Algebra 2<br>Prentice Hall ©2009 |
| Option 2: | | | |
| Course Title:<br>**Algebra** | Course Title:<br>**Geometry** | Course Title:<br>**Advanced Algebra** | Course Title:<br>**Precalculus** |
| Text: Algebra 1<br>Prentice Hall © 2009 | Text: Geometry<br>Holt McDougal, ©2012 | Text: Algebra 2<br>Prentice Hall © 2009 | Text: Algebra and Trigonometry 8th Ed.<br>by Sullivan<br>Pearson Prentice Hall ©2008 |
| Option 3: | | | |
| Course Title:<br>**Geometry** | Course Title:<br>**Advanced Algebra** | Course Title:<br>**Precalculus** | Course Title:<br>**AP Calculus** |
| Text: Geometry<br>Holt McDougal, ©2012 | Text: Algebra 2<br>Prentice Hall © 2009 | Text: Algebra and Trigonometry 8th Ed.<br>by Sullivan<br>Pearson Prentice Hall ©2008 | Text:<br>Calculus 5th Edition<br>Hughes-Hallett<br>Wiley ©2011 |
| Option 4: | | | |
| Course Title:<br>**Geometry D** | Course Title:<br>**Advanced Algebra D** | Course Title:<br>**Precalculus D** | Course Title:<br>**AP Calculus D** |
| Text: Geometry<br>McGraw-Hill ©2012 | Text: Algebra 2<br>Prentice Hall © 2009 | Text: Precalculus 2nd Ed.<br>Prentice Hall ©2001<br>By Sullivan and Sullivan | Text: Calculus 5th Edition<br>Hughes-Hallett<br>Wiley ©2011 |
| Recommendation: Completed Algebra D with C+ Or better &/or teacher recommendation | Recommendation: Completed Geometry D with C+ or better &/or teacher recommendation | Recommendation: Completed Adv. Alg. D with C+ or better &/or teacher recommendation | Recommendation: Completed Precalculus D with C+ or better &/or teacher recommendation |

*Note:* **When in doubt concerning two different placements, the student should always be placed in the more rigorous course.**

*These represent “typical” sequences of district-wide supported courses. Individual high schools may offer unique courses. All placements should be based on individual student interest and need.

**High School Course Descriptions**

*Algebra and Algebra Block*

Algebra is the first course in the traditional college preparatory sequence. Course topics include equation solving, linear sentences, linear inequalities, lines, slope, graphing, exponents and powers, polynomials, systems of equations, quadratic equations, functions, and statistics. Algebra Block is a double period course for students who require additional time to master the objectives.

Text: *Algebra 1, Prentice Hall © 2009.*

*Geometry Plus, Geometry, and Geometry D*

This is a traditional plane geometry course. Topics include reasoning and proof, lines, triangles, quadrilaterals, transformations, similarity, right triangles, circles, area and volume. The differentiated course studies additional topics in greater depth than the regular course. The differentiated course is open only to students who completed differentiated algebra with a recommended grade of C+ or better. The differentiated course is open to other students with permission of the math department chair or designee. Geometry is a one-year course and Geometry Plus is a three semester course for students who require additional time to master the objectives. In addition, Geometry Plus also focuses on reinforcing and developing algebra skills consistent with testing guidelines.

Text Geometry D: *Geometry,* ©2012 McGraw-Hill.
Text Geometry & Geometry Plus: *Geometry*, ©2012 Holt McDougal

*Elements of Advanced Algebra*

Elements of Advanced Algebra is a one semester course that follows the third semester of geometry plus. Course content includes an introduction to advanced algebra concepts in addition to preparation for the NeSA-M. The course uses district created materials. The course is open to a limited number of students who fail first-semester advanced algebra based on teacher recommendation.

*Advanced Algebra and Advanced Algebra D*

Practical applications are the basis for the study of linear equations, inequalities, and functions, systems of equations and inequalities, matrices, quadratic functions, polynomial functions, powers, roots and radicals, exponential and logarithmic functions, rational equations, quadratic relations, sequences and series, and probability. The differentiated course studies additional topics in greater depth than the regular course. The differentiated course is recommended for students who completed differentiated geometry with a grade of C+ or better. The differentiated course is open to other students with permission of the math department chair or designee. The differentiated course qualifies for a weighted grade.

Text Advanced Algebra/Advanced Algebra D: *Algebra 2*, *Prentice Hall © 2009.*
*Precalculus and Precalculus D*

Precalculus is designed for students intending to continue their study of mathematics in the traditional calculus sequence. This course includes a thorough study of trigonometric functions and their properties, limits of functions and sequences, properties of algebraic, exponential and logarithmic functions, and polar coordinates. The differentiated course is open only to students who completed differentiated advanced algebra with a grade of C+ or better. The differentiated course is open to other students with permission of the math department chair or designee. The differentiated course qualifies for a weighted grade.

Text Precalculus/ Precalculus D; *Algebra and Trigonometry Eighth Edition, Pearson Prentice Hall © 2008*

*AP Calculus and AP Calculus D*

Calculus is a college-level course covering topics that are normally studied during the first two semesters of calculus in college. Topics include limits, continuity, derivatives, the definite integral, trigonometric and exponential functions, vectors, methods of integration, and applications of the derivative and integral. The differentiated course completes the first two semesters of college calculus; the regular course completes the first semester and part of the second semester. Students who complete the differentiated course may enroll in the third semester at UNL. Students who complete the regular course may enroll in the second semester course at UNL. Students may qualify for university credit. The differentiated course is open only to students who completed differentiated precalculus with a grade of C+ or better. The differentiated course is open to other students with permission of the math department chair or designee. Both courses qualify for a weighted grade.

Text: Calculus: *Calculus*, University, Wiley © *2011*

*AP Statistics*

AP Statistics is a year-long course emphasizing distributions, data displays, probability, and inferential statistics for decision-making. The prerequisite course is advanced algebra. AP Statistics may also be taken along with Precalculus or Calculus.

Text: *STATS: Modeling the World*. ©2010, Addison Wesley

**High School Math Failure Recommendations* – Semester 1**

| Course Failed | Second Semester Course Options | Comments |
|---|---|---|
| Algebra Block Semester 1 | Off semester Algebra with an algebra support class | Appropriate for students who were close to passing, but need to shore up certain skills. |
| | Off semester algebra block (if available) | Appropriate for the majority of failures. |
| | Continue in second semester algebra block. | Appropriate for students who understand the content but failed for non-academic reasons. Passing second semester can replace first semester failure with "S." |
| | Sit out math second semester and re-enroll next year. | Appropriate for students with severe behavior/non-attendance issues. |
| | | |
| Algebra Semester 1 | Off Semester Algebra | Appropriate for students who lack skills. |
| | Continue in course | Appropriate for students who understand the content but failed for non-academic reasons. Passing second semester can replace first semester failure with "S." |
| | Algebra Block Semester 2 | Appropriate for students with a weak understanding of the content, but who failed primarily for non-academic reasons. Passing second semester can replace first semester failure with "S." |
| | | |
| Geometry Plus 1 Semester 1 | Continue in course. | Only option. Feasible because the semesters are largely independent of each other. |
| | | |
| Geometry Plus 2 Semester 1 | Continue in course. | Only option. Critical for NeSA-M preparation. |
| | | |
| Geometry Semester 1 | Off Semester Geometry. | Appropriate for majority of failures. |
| | Geometry Plus 1 Semester 2 | Appropriate for students who need more support and processing time to be successful. |
| | | |
| Advanced Algebra Semester 1 | Elements of Advanced Algebra | Appropriate for juniors in advanced algebra (prepares them for NeSA-M). Can repeat Advanced Algebra as a senior. |
| | Off Semester Advanced Algebra (if available) | Appropriate for majority of failures. |
| | Non-AP Stat Course | Appropriate for seniors. |
| | | |
| PreCalculus Semester 1 | Precalculus Semester 2 | Appropriate for all students. |
| | | |

### High School Math Failure Recommendations – Semester 2

| Course Failed | Fall Semester Course Options | Comments |
|---|---|---|
| Algebra Block Semester 2 | Off semester 2 Algebra with an algebra support class | Appropriate for students who were close to passing, but need to shore up certain skills. |
| | Off semester algebra block. | Appropriate for the majority of failures. |
| | Continue to Geometry Plus. | Appropriate for students who have 10 hours of math credit from Algebra Block Semester 1. |
| | | |
| Algebra Semester 2 | Off Semester 2 Algebra | Appropriate for students who lack skills. |
| | | |
| Geometry Plus 1 Semester 2 | Continue in course. | Appropriate for students who have 20 hours of credit from Algebra block and credit in Geometry Plus Semester 1. |
| | Repeat in fall if the student also failed Geometry Plus Semester 1 | Appropriate for students who did not enter Geometry Plus with 20 hours of credit from Algebra Block. |
| | | |
| Geometry Plus 2 Semester 2 | | Placement dependent on whether credits are needed for graduation. |
| | | |
| Geometry Semester 2 | Off Semester Geometry. | Appropriate for majority of failures. |
| | Geometry Plus 2 Semester 1 | Appropriate for students who need more support and processing time to be successful. |
| | | |
| Advanced Algebra Semester 2 | Off Semester Advanced Algebra (if available) | Appropriate for majority of failures. |
| | Start Advanced Algebra over again in fall. | Appropriate for juniors who failed due to a lack of skill. |
| | | |

*These are general recommendations. Each student is unique and must be treated accordingly. Carefully examining a student's current math credits, prior course work, individual needs, and post-secondary plans, all play a role in individual recommendations. Exceptions can and should be made as appropriate for individual students.

**Assessment and Reteaching (Middle Level and High School)**

Re-teaching and re-learning is a research-based instructional strategy to improve student learning. The National Mathematics Advisory Panel recommended the use of assessments for the purpose of modifying instruction based on student progress (NMAP, 2008). Research indicates that when the results of assessment are used to provide students additional instruction, practice, and reinforcement in the skills and/or concepts with which they struggle, that student achievement is improved (Baker, Gersten, & Lee, 2002). Research also indicates that when students who struggle to learn mathematics are allowed enough time to master content, they can perform at levels approaching high achieving students (Usiskin, 2007). The research clearly supports the use of additional instructional time as a strategy to improve student achievement (Balfanz, Mac Iver, & Byrnes, 2006; Burris, Heubert, & Levin, 2006; Cawelti & Protheroe, 2001). Marzano (2010) has found that re-teaching and re-assessment can result in a twenty-percentile improvement in student achievement. *Therefore, re-teaching and re-assessment is now a required district expectation in mathematics from Kindergarten through Differentiated Precalculus*. In order to prepare students for possible course work beyond high school, there is a steady decline in the amount of re-assessment that takes place as students progress through the curriculum as outlined below:

- Algebra: Retesting occurs on both chapter tests and cumulative assessments. Retesting is used for grade replacement.
- Geometry: Retesting occurs only on cumulative assessments. Retesting is used for grade replacement.
- Advanced Algebra: Retesting occurs only on cumulative assessments.
- PreCalculus: Retesting occurs only on first and third quarter cumulative assessments.
- Calculus: No retesting.

**Format of District Assessments**

The NeSA-M emphasizes the application of mathematics. Specifically, many of the assessment items on the NeSA-M require students to draw on a variety of mathematical topics and apply these topics to novel problems that students may have never previously seen. This places an emphasis on student mathematical understanding, as opposed to rote procedural skill, so that students can make appropriate connections among mathematical topics and draw on their understanding to solve problems. Therefore, all secondary math objectives and assessments through precalculus are organized around connected "Big Ideas." It is a district expectation that district chapter (Big Ideas), cumulative assessments, and finals will be used to ensure consistency and that they will not be modified (beyond special education requirements) to adjust the cognitive demand.

The purpose of Big Ideas is to emphasize mathematical connections and understanding, not discrete and disjoint topics. For example, previously in algebra there were separate objectives on solving systems of linear equations by graphing, using substitution, and using elimination. These objectives would have typically been taught as discrete topics, as opposed to three equally valid strategies to solve a system of linear equations. The topics would then have been assessed separately, and the individual topics would have been re-taught and re-assessed separately if necessary. Now, the Big Idea is solving a system of linear equations. Instructionally, the emphasis is on solving a system, students learn three different, but equally valid strategies to do this, and when assessed, students are simply presented with a system to solve. The choice of strategy is up to the individual student. It is not necessary to assess all three strategies individually.

In order to address teacher workload concerns, math assessments have embedded the re-assessment concept within chapter assessments (algebra only) and on periodic cumulative re-assessments. The use of these district assessments and embedded re-assessments removes the need for teachers to create re-assessments of their own and ensures that all students have an opportunity to be re-assessed. Embedding the re-assessment in formal subsequent assessments also lessens the tendency of some students to "game the system" and simply wait until after an initial assessment has been administered to determine what will be assessed and then study those specific items for the re-assessment.

*Re-assessment Procedures*

Re-teaching (not just additional practice) and re-learning must take place prior to re-assessment for the strategy to be effective at improving student learning. It is assumed that all teachers will provide all students an opportunity to continue to master topics they have not yet mastered through appropriate re-teaching either inside or outside of the regular instructional time. Because re-assessment is embedded within formal and required district assessments, all students will have access to re-assessment opportunities regardless of their compliance with behavioral expectations, e.g. homework completion.

Re-assessment on Chapter Assessments

- Chapter tests which do not begin a semester or follow a cumulative assessment will include a re-assessment of <u>one</u> big idea from the previous chapter (applies to algebra and algebra block). This is determined in part by what is assessed by NeSA-M and in part based on what is most essential for students to know moving forward in the curriculum.
- Assessment sections (Big Ideas) from chapter tests will be entered in the district gradebook as individual scores.
- The length of the re-assessment section may not match the original assessment.
- A score is replaced only if the student improves because re-assessment is more likely to prove a positive than a negative.
- Although the opportunity to re-learn should be a significant motivator, the reality is that for many students the opportunity to replace a low score serves as a motivation to engage in the re-learning process.
- Because retention and understanding are essential goals, the re-assessment section of a chapter test or cumulative assessment is used not only for the purpose of re-assessment and grade replacement, but also constitutes a new score in a student's grade.

Cumulative Assessments

- Cumulative assessments are used as a way to emphasize the importance of retention as well as an additional vehicle for students to demonstrate re-learning.
- Cumulative assessments are administered periodically following 2-3 chapters of instruction with the exception of Geometry Plus.
- Cumulative assessments will assess all big ideas from the previous chapters (since the time of the last cumulative assessment).
- In algebra and above, the last chapter of each semester will not be included on a cumulative assessment. This provides teachers with additional time to process cumulative exams and create instructional space prior to the final exam.
- Cumulative assessments focus on assessment of students' retention of critical concepts. The rigor of a cumulative assessment resides in the retention. It is not possible to re-assess every concept as it was assessed on the original assessment, nor at its original depth.
- The cumulative assessment also counts as a single new score in a student's grade. This serves to motivate students to retain previous learning.

Final Exam

- The final exam serves to assess student learning across a semester's big ideas, but can only survey essential topics – it cannot re-assess every big idea at the level done on individual chapter tests.
- Although the final exam is not used to replace previous individual assessment scores, a teacher can consider a student's performance on the final for a student's final grade (if that does not lower a student's final grade).
- The goal is for the final exam to account for approximately 10% of a student's grade.

District Common Assessments

- District Common Assessments (DCAs) are a critical component of the district's NeSA-M preparation program. DCAs must be administered in designated courses at designated times. Results must be used to plan intentional review and re-teaching of critical NeSA objectives.

Additional Assessment Guidelines

- All summative tests in classes other than Algebra Block are to be completed in one 50-minute period. The reassessment portion of an assessment may be given on a separate day, but should be limited in time (approximately 20 minutes). When given on a separate day, no review should be done prior to the reassessment. Algebra Block classes may continue summative assessments into a second period on the same day as needed.
- Teachers should use their professional judgment concerning providing individual students additional time to complete assessments. For example, if a student works diligently to complete an assessment, but needs some additional time to complete it, the

teacher can and should allow the student to come in before or after school to complete it.

- Geometry Plus will give a summative exam following each Big Idea in a unit. Exams will include the same content as Geometry and have additional questions to reinforce the algebra concepts included in geometry. The cumulative exam will cover two Big Ideas and is to be given after reteaching of both Big Ideas in the unit. Each exam should be completed in one 50 minute period. The NeSA-M Reference Sheet may be used on all assessments in Geometry Plus.
- The NeSA-M Reference Sheet may be used on cumulative and final assessments in middle level courses, Algebra, Geometry, and Advanced Algebra courses (regular and diff).
- The content of summative assessments must remain consistent throughout the district. Only changes in layout of the assessment are permissible.
- Summative assessments are not to be given by separating Big Idea sections into separate portions unless the student is permitted additional time.
- Cumulative assessments serve two purposes: to measure retention and to serve as a reassessment following re-teaching (grade replacement). Therefore it is critical that students have the opportunity to re-learn concepts they did not master on previous Big Idea tests prior to administering the cumulative assessment. Shutting down new instruction to spend multiple days reviewing for a cumulative assessment is not appropriate; rather, retention can and should be promoted through intentionally designed and teacher provided warm-up activities that review critical concepts while continuing new content.
- All summative assessments, with the exception of the final exam, also serve a formative function in the sense that teachers should use the results to design and provide students with appropriate re-teaching prior to the next summative test or cumulative assessment.
- In either a makeup situation or a situation where a student is allowed additional time, it is never appropriate for a student to return and complete a page which has previously been viewed.

Synergy (Edupoint Gradebook)

- Point values for Big Ideas, Cumulative Assessments, and Finals, are found on DocuShare and must be followed for student grading to be consistent and equitable across the district.
- At all grades and in all courses, the grading template is 80% summative and 20% formative.
- Only chapter/Big Ideas, cumulative assessments, and finals count in the summative category.
- Quizzes and homework count in the formative category.
- Quizzes should generally have a point value between 25 and 50 points, and all teachers of a same course within a building should assign the same point value to quizzes.
- Homework should count between 5 and 10 points daily. Homework should be collected and "processed" (examined in order to guide future instruction) daily for correctness; it should not be graded based on completion.

**The “Arc” of an Effective Math Lesson: The 5Es**

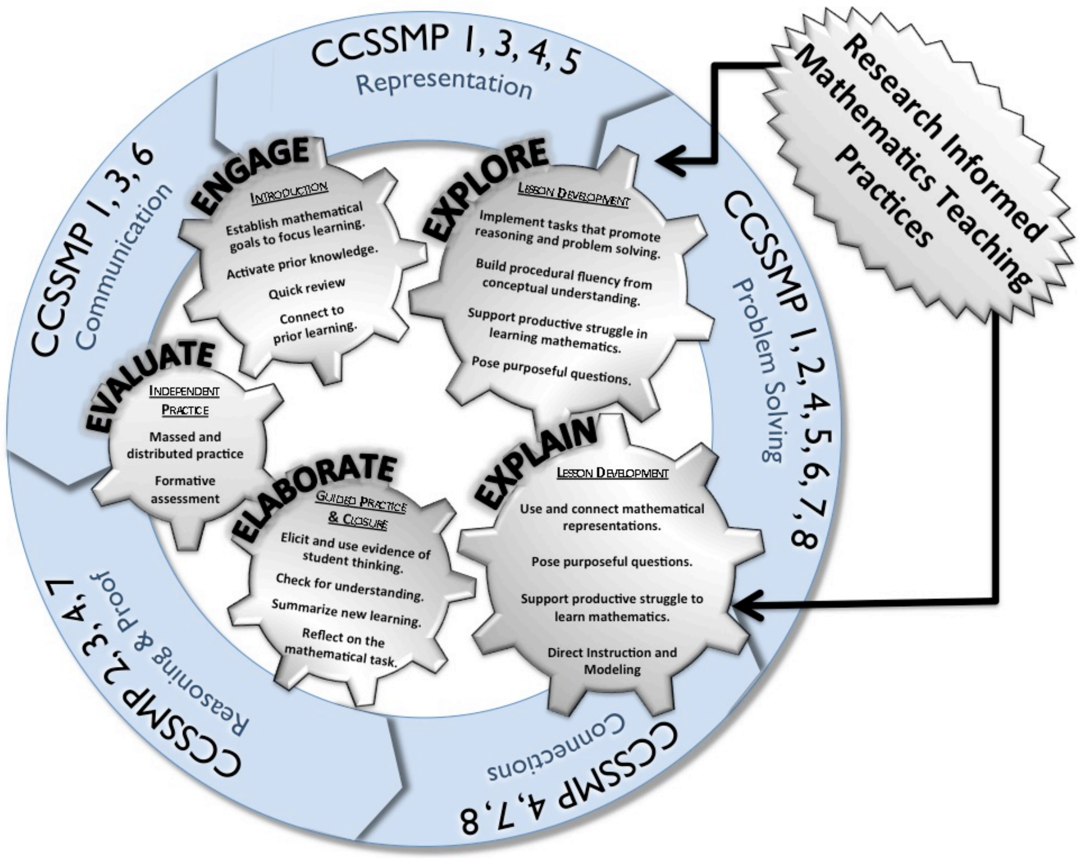
Research on effective mathematics teaching has not suggested a direct association between a single method of teaching and a resulting goal. Research points to certain features and “high-leverage practices” that result in improved student learning (Hiebert & Grouws, 2006; NCTM, 2014). Research-informed instructional strategies, those that have a positive impact on student learning, must be situated within the overall “arc,” or learning cycle of a mathematics lesson. The arc of an effective mathematics lesson, what unfolds from the beginning to the end of a lesson has five major components (Bybee et al., 2006; Dixon et al., 1998; Muijs & Reynolds, 2000; Weiss, Heck, Shimkus, 2004; Whitehurst, 2003):

- Engage
- Explore
- Explain
- Elaborate
- Evaluate

Collectively these five components are referred to as the 5E Instructional Framework. The 5E Instructional Framework is the common LPS curriculum-wide instructional framework. The 5E Instructional Model provides a common framework for lesson planning and common vocabulary that teachers, instructional coaches, and administrators can use as they collaboratively design lessons and reflect on the effectiveness of implemented lessons and instructional strategies. The concept behind the 5E Instructional Model is to begin with students’ current knowledge, make connections between current knowledge and new knowledge, engage students in worthwhile learning tasks, make mathematical connections and provide direct instruction as necessary of the ideas students would not be able to discover on their own, provide opportunities for students to demonstrate their understanding, and provides teachers the opportunity to respond to student needs based on students’ emerging understanding.

It is important to understand that these major components of effective math lessons do not necessarily have to be done in lock step order (although obviously the Engagement Phase marks the start of a mathematics lesson). The graphic below illustrates that these components interact in ways as different lessons unfold, i.e. teachers may move between these components and the components interact fluidly during a math lesson. The sizes of the circles represent the relative time spent on each component, e.g. the Exploration and Explanation phases consume the bulk of a typical math lesson. The circles are depicted by “gears” to represent their interaction.

**A Visual Representation of the Components of an Effective Mathematics Lesson**




*Component 1: Engage* (Introduction)

Every math lesson has to have a starting place and it begins by engaging students. The purpose of the Engagement Phase is to activate prior knowledge, review critical prior learning, and introduce the lesson's objective. Daily routines, or an opening problem are effective ways to begin a math lesson, provided the daily routines or problem are connected to the important mathematics to be learned or maintained. An opening problem (or problems) might make connections to pre-requisite knowledge necessary for the new lesson, provide spiral review to support retention, provide critical NeSA-M review based on DCA results, or be based on a review of the previous day's independent practice, e.g. a problem that reviews a concept that many students misunderstood that needs clarification.

The Engage Phase of the lesson also includes a clear statement of the lesson's learning objective to students in student friendly language. Sometimes the learning objective will be shared after the Explore Phase of the lesson because the lesson is intentionally designed to have students "discover" the objective as a result of the Explore task.

*Component 2: Explore* (Lesson Development)

During the Explore Phase of the lesson students individually or collaboratively work on a mathematical task designed to help them develop a deep understanding of the mathematical skill or concept that is the focus of the day's learning objective.

A well-designed task engages students with the mathematical concepts by having students interact purposefully with the content (Weiss, Heck, & Shimkus, 2004). A mathematical task is often a rich problem that has been intentionally selected to lead students to the mathematical concepts that are the focus of the day's lesson. Worthwhile mathematical tasks are "ones that do not separate mathematical thinking from mathematical concepts or skills, that capture students' curiosity, and that invite them to speculate and to pursue their hunches" (NCTM, 1991, p. 25). In order for students to build mathematical understanding, the tasks with which they engage "must allow the students to treat the situations as problematic, as something they need to think about rather than as a prescription they need to follow" (Hiebert et al., 1997, p. 18). Effective tasks also require students to "think," i.e. they are cognitively demanding (outlined further below). High cognitive demand tasks provide students opportunities to explain, describe, justify, compare, or assess; to make decisions and choices; to plan and formulate questions; to exhibit creativity; and to work with more than one representation in a meaningful way (Silver, 2010).

*Component 3: Explain* (Lesson Development)

The Explain Phase involves presenting information that students are unlikely to discover on their own. This phase provides the teacher the opportunity to address mathematical issues/concepts that students might miss during the Explore phase, clarify and help students make important mathematical connections, draw students' attention to efficient solution strategies, and clear up any misconceptions that may have developed. The Explain phase is the teacher's opportunity to help students make connections between the task they do in the Exploration phase and connect concrete representations, and the language and symbols of mathematics. Physical materials are not automatically meaningful to students and need to be connected to the situations being modeled (NRC, 2001). In all explanations it is important to link the math drawing or other visual support to the formal math method for each step of that method. It is this tight linking that enables the meanings for the visual or contextual supports to become attached to the formal math method and notations, and thus advance student understanding (Fuson & Murata, 2007). It is essential that teachers, through purposeful questioning to generate discussion, as well as written materials, support students in making the explicit construction of links between representations and the related symbol procedures (Ma, 1998).

It is important to be clear: The Explore phase of the lesson does not mean that teachers should never tell or direct students. Telling is often necessary and appropriate. Appropriate telling can include sharing mathematical conventions, suggesting alternative solution methods, introducing more clear and efficient recording techniques, and articulating ideas in students' solution methods. "Telling is legitimate if it does not take fundamental agency for making sense away from students" (Hiebert et al., 1997).

The research is clear: "… empirical research has provided overwhelming and unambiguous evidence that minimal guidance during instruction is significantly less effective and efficient than guidance specifically designed to support the cognitive processing necessary for learning" (Krischner, Sweller, & Clark, 2006). The purpose of the Explanation phase is to ensure that students receive the instructional "guidance" necessary for them to meaningfully process tasks completed during the Explore phase. Without the Explanation phase and the relaying of content knowledge, many students – particularly weaker students – will not benefit from the lesson or activities (the Explore Phase), no matter how engaged they were.

*Component 4: Elaborate* (Guided Practice and Closure)

The Elaborate Phase provides students an opportunity to deepen their understanding of the mathematics by engaging in guided practice activities. Guided practice is an essential component of effective lessons. Fuson (2003) has pointed out that helping students build initial correct methods is much easier than correcting errors. Carefully designed guided practice and support during learning are important aspects of developing mathematical proficiency. It is during this phase of guided practice that teachers provide students with scaffolded support as students gradually assume more independence. Practice is important, but effective practice is supported by careful monitoring and instructional support that is focused on students learning mathematics with understanding (Fuson, 2003).

Contrary to popular belief, practice does not make perfect, it makes permanent. It is important not to rush to independent practice because repetitive practice will "freeze" a student at his/her current level of understanding (Brownell, 2007). As students engage in guided practice, teachers carefully monitor student understanding, and make appropriate instructional adjustments. For example, if it is clear that students are struggling with a particular guided practice problem or concept, then re-teaching and clarification can take place. The Elaborate phase should always end with a formal "Check for Understanding" (formative assessment), followed by additional re-teaching as necessary, before the lesson is summarized.

The Elaborate phase of the lesson ends with lesson closure/summary. Lesson closure is another opportunity to help students recognize the key ideas in a lesson and make connections. Research indicates that teachers in Japan summarize lessons, and summarize key points during lessons, more than teachers in any other country (NCES, 2003). Student participation in the closure process is critical to their assimilating and gaining a true understanding of the lesson. Students must be active agents in analyzing, summarizing, and connecting what they have just learned (Wolf & Supon, 1994).

*Component 5: Evaluate* (Independent Practice)

The Evaluate phase is a formal formative assessment opportunity. When students are assigned independent practice (homework), and teachers collect that work and process it on a daily basis, the results can be used to provide teachers information about student understanding and performance, and used to guide future lesson development, e.g. the choice of the opening task in the Engagement phase of the following lesson.

Independent practice is critical to student learning of mathematics. "Nothing flies in the face of the last 20 years of research more than the assertion that practice is bad. All evidence, from the laboratory and from extensive case studies of professionals, indicates that real competence only comes from extensive practice. In denying the critical role of practice one is denying children the very thing they need to achieve real competence" (Anderson, Reder, and Simon, 1998). "It is an error to insist, to quote some, that 'there is no place for drill in the modern conception of teaching.' True, there is no place for unmotivated drill on ill-understood skills; but the statement goes too far in saying that there is no place at all for repetitive practice" (Brownell, 2003).

Practice only drifts into the area of mindless drills when students practice things they do not understand. If the *Elaborate* phase of the lesson was effective, then independent practice is not only effective, but the amount of independent practice necessary to ensure mastery is actually reduced. It is only when students practice procedures they do not understand that they need extensive practice so as not to forget the steps (NRC, 2001). If students seem to need massive amounts of practice in order to learn something, that is more often than not an indicator that the instruction was insufficient to generate the level of initial understanding necessary to make independent practice effective.

**Homework**

Homework is a critical way to provide independent practice. Students at every grade level should have appropriate daily mathematics homework. Homework can be effective in improving students' achievement on school-based assessments. In a recent summary of the research, Cooper (2008) found a positive relationship between the amount of homework that students do and their achievement in mathematics. Short practice assignments were most effective in the elementary grades, up to 90 minutes of homework were most effective in the middle grades (total time for all subjects), and up to two hours were most effective in high school (total time for all subjects). Students need more than just massed practice. Distributed practice over time with feedback promotes student retention and transfer of knowledge (Pashler et al., 2007). Another finding from the research is that homework/independent practice is most effective when teachers provide feedback to students' homework on a daily basis and give students written descriptive feedback that goes beyond simply marking their work as correct or incorrect (Davies, 2007; Marzano, 2006; and Shuhua, 2004). In other words, "completion" grades on homework are insufficient because such feedback does not provide students with an indication of what they understand, don't understand, and how to proceed to improve their understanding. Developing consistency with respect to homework procedures and guidelines is a constructive topic for collaborative teamwork.

The majority of instructional time at school should be devoted to new instruction and guided practice. District policy 6550 addresses homework and states that "School homework should be related to the curricular objectives and be consistent with research guidelines. The assignment of homework is encouraged to aid student learning."

With district policy and research findings in mind (Sutton & Krueger, 2002; Marzano, Pickering, & Pollock, 2001; O'Connor, 2002), the district makes the following recommendations with respect to mathematics homework:

- Homework is independent practice of skills already learned in school. The purpose of independent practice is the development of mastery. It is essential that significant guided practice with feedback and corrective instruction be completed prior to assigning independent practice. The amount of guided practice required prior to assigning independent practice will vary according to the difficulty of the concept/skill.

- **It is a district expectation that students in grades 6-12 enrolled in a mathematics class will have homework assigned nearly every day.**

- The fact that some students will not complete and return homework is not an excuse for not assigning independent homework. Failure to assign homework because some students will not return it sets an artificially low expectation for all students.

- It is important to remember that homework is most effective when it is promptly commented on by the teacher and the results used to guide future instruction. It is a waste of precious instructional time to "check" or "grade" student papers in class.

- How do I grade homework? The important thing about homework is that teachers collect it, evaluate it, and use it to guide instruction. Students do need some form of timely feedback to make the work meaningful. Teachers may choose to spot check a subset of critical problems, i.e. it is not an expectation that every homework problem will be corrected everyday. However, it is an expectation that teachers will collect and process homework daily. Homework is a formative activity designed to foster student mastery.

**Mathematical Processes and the Standards for Mathematical Practice**

Around the outside of the 5Es (see the diagram on p. 26) are the five NCTM (2000) process standards as well as the Common Core State Standards for Mathematical Practice. These process standards represent how students are to experience the mathematics and develop their understanding and proficiency:

- *Problem Solving*. Problem solving is an integral part of all mathematics learning and therefore should not be isolated from the content. Emphasizing problem solving means that students have opportunities to build mathematical knowledge through problem solving; that students solve problems that arise in mathematics and in other contexts; that students apply and adapt a variety of appropriate strategies to solve problems; and that students monitor and reflect on the process of mathematical problem solving.

- *Reasoning & Proof*. Being able to reason is essential to understanding mathematics. Emphasizing reasoning and proof means that students have opportunities to make and investigate mathematical conjectures and develop and evaluate mathematical arguments.

- *Communication*. Communication is an essential part of students' mathematics education and is the primary means by which children share and clarify their understanding. Emphasizing communication means that students have the opportunity to organize and consolidate their mathematical thinking through communication; that students communicate their mathematical thinking coherently and clearly to peers, teachers, and others; that students analyze and evaluate the thinking and strategies of others; that students use the language of mathematics to express mathematical ideas precisely.

- *Connections*. When students can connect mathematical ideas, their understanding is deeper and more lasting. Emphasizing connections means that students have opportunities to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics.

- *Representation*. The ways in which mathematical ideas are represented is fundamental to how students understand and use those ideas. Emphasizing representation means that students have opportunities to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret mathematical phenomena.

The process standards are linked to the Common Core State Standards for Mathematics [CCSSM] Standards for Mathematical Practice. The Standards for Mathematical Practice represent ways in which students are to engage with the content standards in order to develop a deep understanding of mathematics and associated habits of mind. Planning to engage students in these practices should be a component of teachers lesson planning. The eight standards for mathematical practice are described below.

1. **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and changes course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculators to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without

necessarily attending to their referents – and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which am argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing

calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 X 8 equals the well remembered 7 X 5 + 7 X 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 X 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back fro an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculators are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

**Behavior, Effort, Participation, and Extra Credit**

Achievement alone – the student's mastery of district math objectives -- should be the basis for his/her grade. Effort, participation, attitude, and other personal or social characteristics are inappropriate components of a grading plan that is based on achievement. Although these factors may contribute to a student's achievement, the interpretation of these characteristics varies widely from teacher to teacher and including any of them blurs a student's true academic achievement and the meaning of a grade.

Similarly, extra credit should not be part of a student's grade as it frequently inflates a student's grade and does not adequately reflect a student's mastery of the district objectives. As Stiggins (2001) has written, "some teachers try to encourage extra effort on the part of their students by offering extra credit opportunities … [however] if grades are to reflect achievement, you must deliver the consistent message that *the more you learn, the better your grade* … to communicate effectively, grades must reflect the amount learned – not how much work was done to accomplish the learning." Therefore, the practice of including "extra credit" in a student's grade is not permitted.

**Effective Use of Instructional Time**

Instructional time is limited and must be carefully used. According to the *1999 TIMSS Video Study of Eighth-Grade Mathematics Teaching*, high performing countries spend more of their instructional time on new material than they do on review. For example, teachers in Hong Kong and Japan spend up to 76% of lesson time on new content. In the United States math teachers typically spend 50% or less of lesson time on new content and the rest reviewing previous content (going over homework).

Effective teachers spend more time discussing new content and less time discussing homework; highly effective teachers seldom if ever spend instructional time correcting homework. Although it is important to give students an opportunity to discuss homework, the amount of time dedicated to this task must be carefully monitored. In order to protect instructional time for new content many teachers correct a sample of homework problems (themselves) and move the discussion of homework to the end of the lesson to protect new content instructional time. Carefully chosen warm-up problems can provide the review necessary to transition directly into new content without discussing homework at the beginning of the lesson.

**Effective use of PLC/Data Team Time**

One of the most significant challenges facing mathematics education in the U.S. is the inconsistency in instructional effectiveness (Loveless, 2012; Morris & Hiebert, 2011). Therefore, it is a district expectation, and teachers have professional responsibility (Stigler & Hiebert, 1999), to collaborate to ensure **consistent expectations** (e.g, grading, homework, testing) and work on instructional strategies and lesson designs. Research indicates that when teachers work together in professional learning communities, utilizing the Data Team process, that collegial support enables individual teachers to consider and revise their classroom practice confidently; that they take more pride in their department and enjoy teaching more; that student expectations become more consistent; and that student learning is positively impacted (Kanold and Larson, 2012; McLaughlin and Talbert, 2001).

It is the Math Curriculum Department's recommendation that secondary math PLCs focus on lesson design and reflection. The Math Step 4 Data Team Guide (located on DocuShare) is a critical tool to guide this process. The essential factor in growth and improvement in teaching is lesson preparation and the analysis of lesson outcomes both during and after each lesson (NCTM, 2007). Each year, each course-based PLC should select 5-10 critical lessons, based on assessment data, and focus on the development of extensive lesson designs for these critical topics. This type of intensive lesson planning is not only a high-leverage instructional strategy, but it also prevents the degradation of PLC time into mere story-swapping and the sharing of materials (Kanold & Larson, 2012).

**Calculator Guidelines**

Technology is changing the ways in which mathematics is used and is driving the creation of new fields of mathematical study. Consequently, the content of mathematics programs and the methods by which mathematics is taught and student learning is assessed are changing. The capacity for the appropriate use of technology to develop, enhance, and expand students' understanding of mathematics is great. A comprehensive mathematics curriculum should help students learn to use calculators and other technological tools. These tools are a part of many aspects of students' education and will be a standard part of how they study mathematics and science and how they do mathematics in the workplace. It would be remiss to not make their use a part of contemporary mathematics education.

At the same time, there are still many mathematical procedures and concepts that students must learn and be able to use without depending upon the use of calculators. Clearly, the availability of calculators does not and should not replace the need for students to learn basic facts, to compute mentally, or to do reasonable paper and pencil computation. For example, while everyone should be able to do simple computations with fractions by hand, techniques for finding square roots by hand are no longer a part of the curriculum. Sound mathematics instruction uses technology to enhance teaching, but does not use it to eliminate the need to teach students to think or to compute. Therefore, the professional question of when to use technology and when not to use it is one that mathematics teachers must face almost daily. In general, the appropriateness of calculator use at all levels is judged in terms of the instructional objective. If the instructional objective is focused on solving problems in context and not the computations embedded within the problem, then the calculator may be an appropriate tool. If the instructional objective is the mastery of a specific skill or procedure, then the calculator is an inappropriate tool.

To further clarify the appropriate use of calculators, the Lincoln Public Schools Mathematics Curriculum Department has established the following guidelines with respect to calculator use.

Grades K-3

The major objective of primary school mathematics is the development of number sense. Number sense evolves from concrete experiences and takes shape in oral, written, and symbolic expression. Students at this level must develop mastery of computational procedures without the use of a calculator and consequently the use of calculators in grades K-3 is a very minor part of the curriculum. Occasionally, teachers may use simple four function calculators with students to explore and experiment with mathematical ideas such as patterns and numerical properties or to permit students to solve problems that involve more complex computations.

Specifically, the district endorses the following practices at the K-3 level:

- It is appropriate for teachers to occasionally use calculators with students to explore and experiment with mathematical ideas such as patterns and numerical properties. This use should support a specific instructional objective and the use of calculators should be under the careful supervision of the teacher. In particular, some modules in the differentiated curriculum will require calculator use.

- Because of the importance of teaching basic facts and computation skills at this level, it is appropriate for teachers to carefully regulate student access to calculators. In most cases, it would be inappropriate for students to have "open" access to calculators at this level.

- Students at this level are not permitted to use calculators on any portion of the ITBS.

- Because calculator use is carefully regulated by the teacher at this level, it is not necessary for every teacher to have a complete classroom set of calculators.

Grades 4-6

In the intermediate grades, instruction with calculators has increased potential to enhance and expand students' understanding of mathematics. However, this instruction must focus on developing students' ability to know how and when to use a calculator. Skill in estimation and the ability to decide if the solution to a problem is reasonable, and skill in selecting the appropriate solution strategy (mental math, paper and pencil, or calculator) are essential adjuncts to the effective use of the calculator. As such, calculators are simply another tool that is made available to students when they are working in a problem solving context and the emphasis is on their ability to interpret a problem situation and determine the procedure required to solve the problem. Calculators at this level should never be used to replace the recall of basic facts and the ability to perform routine computational procedures either mentally or with paper-and-pencil.

Specifically, the district endorses the following practices at the 4-6 level:

- It is appropriate for students to use calculators under teacher direction to explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties, and functions.

- It is appropriate for students to use calculators under teacher direction when focusing on problem-solving processes rather than the tedious computations that often develop when working with real data in problem situations.

- It is appropriate for students to use calculators under teacher direction for the purpose of gaining access to mathematical ideas that go beyond those levels limited by traditional paper-and-pencil computation.

- It is a district expectation that students will be able to perform basic computational procedures without the aid of a calculator. This includes:

Grade 4

- Recall of basic multiplication facts through 12.
- Three digit addition and subtraction.
- Decimal addition and subtraction to the hundredths place.
- Up to a 3 digit number times a 2 digit number.
- Division with a 1 digit divisor.
- Addition and subtraction of fractions with a common denominator.

Grade 5

- All grade 4 skills.
- Four digit addition and subtraction.
- Addition of fractions without a common denominator.
- Decimal division with a dividend to the tenths place.
- Division with a two-digit divisor.
- Multiplication involving powers of ten.
- Decimal multiplication with one factor to the hundredths place.

Grade 6

- All grade 5 skills.
- Decimal division with dividend to the hundredths place.
- Addition of mixed numbers.
- Multiplication of fractions.
- The quotient of two fractions.
- Decimal addition and subtraction to the thousandths place.
- Decimal multiplication

Grades 7-8

Grades seven and eight represent transition years with respect to the use of calculators. In these grades it is still expected that students will carry out certain procedures without the use of a calculator. However, by eighth grade the curriculum no longer focuses on computational skills and shifts to preparation for algebra and geometry. Therefore, the calculator can serve as a tool that may give students who have not yet mastered certain skills access to upper level mathematics.

It is a district expectation that students will be able to perform basic computational procedures without the aid of a calculator. This includes:

Grade 7

- All grade 6 skills.
- Decimal addition and subtraction to the hundredths place.
- Product of a mixed number and a whole number.
- Product of two mixed numbers.
- Quotient of a mixed number or simple fraction divided by a whole number.
- Decimal multiplication (thousandths place times the tenths place).
- Finding the percent of a number.

Grade 8

- All grade 7 skills.
- Decimal division (hundredths place dividend, tenths place divisor).

In addition, the following guidelines should be kept in mind:

- It is appropriate for students to use calculators to explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties, and functions.

- It is appropriate for students to use calculators when focusing on problem-solving processes rather than the tedious computations that often develop when working with real data in problem situations.

- It is appropriate for students to use calculators under teacher direction for the purpose of gaining access to mathematical ideas that go beyond those levels limited by traditional paper-and-pencil computation.

- The primary objective of seventh and eighth grade mathematics is not the mastery of computational skills. Some periodic practice of previous skills and those listed above is appropriate, but should not become the dominant feature of the curriculum.

- Students in algebra should have routine access to calculators, except with the objective is focused on computation, for example simplification of radicals.

- For some students whose computational ability is limited, tools such as calculators assist students in studying challenging mathematics despite their difficulty with computation. Therefore, it is appropriate for students with special needs and others to have access to calculators so that their opportunity to study higher level mathematics is not restricted.

- Teachers of mathematics courses at all levels have the right to restrict the use of calculators in their classroom in order to ensure that students understand certain mathematical concepts and are able to perform routine symbolic procedures.

Grades 9-12

In grades 9-12, the math curriculum focuses on approaching problems numerically, algebraically, and graphically. Students are encouraged to use these different approaches to examine problems from different perspectives and find multiple methods to solve problems. In order to approach problems both numerically and graphically, students may make extensive use of hand held graphics calculators.

Although it remains important for students to be able to perform routine computations both mentally and with paper and pencil, at the high school level students will also make extensive use of scientific calculators throughout the curriculum. This practice gives all students access to upper level mathematical concepts.

Specifically, the district has established the following practices at the high school level:

- The use of laptop computers or calculators with algebraic capacity (e.g., TI-92, TI-89, or TI Voyager) is not permitted on tests and quizzes unless the classroom teacher announces a policy that permits their use in some circumstances. This restriction is usually necessary because the use of this technology often permits a student to find answers to many problems without understanding the underlying mathematical concepts or processes.

- In geometry, students and teachers may use the TI-92 as an interactive and dynamic environment for the investigation of geometric properties and relationships. The Geometer's Sketchpad or GeoGebra serves a similar function.

- Students in algebra courses and courses that build on algebra should have access to graphing calculators, although it is not required that they purchase their own calculator.

- Students in precalculus and calculus will make extensive use of graphing calculators and the district provides students with this technology. Students are not required to purchase their own calculator. The recommended model is the TI-83+.

- Teachers of mathematics courses at all levels have the right to restrict the use of calculators in their classroom in order to ensure that students understand certain mathematical concepts and are able to perform routine symbolic procedures. For example, precalculus teachers may assess student knowledge of trigonometric values without the aid of a calculator and calculus teachers may use the UNL Gateway Exams to assess student knowledge of derivatives and integrals without the aid of a calculator.

- High school students are permitted to use certain calculators on the tenth grade PLAN (pre-ACT) test, the ACT, the SAT, and the UNL Math Placement Exam.

  The ACT, PLAN, and AHSME exclude the use of the TI-89.

  All tests exclude the use of the TI-92.

**Multicultural Connections**

It is important that students appreciate mathematics in its historical context (NCTM, 1989). Students should be aware of the importance of mathematics throughout history, and they should have a sense of the evolution of mathematical thought over time (Gardella, et al., 1992). Most importantly, students should come to understand that the body of mathematical knowledge we have today is not the work of a select few, but rather the result of a vast and culturally diverse group of men and women from all around the world (Gardella, et al., 1992). Learning about the contributions of various cultures to the development of mathematics can lead students to a better understanding of mathematical concepts, as well as an appreciation for the cultures involved (Gardella, et al., 1992).

The district has outlined 18 student proficiencies with respect to multicultural education. Mathematics instruction emphasizes the first of these 18 proficiencies. This proficiency states that “students will know the histories, cultures, and contributions of African Americans, Asian Americans, European Americans, Hispanic Americans, and Native Americans.” To support this proficiency, it is a major outcome of each 9-12 mathematics course that “students will know and value the contributions to mathematics made by persons of diverse racial, ethnic, and cultural backgrounds.”

Although the mathematics textbooks in use include many cultural and historical references, an audit of the curriculum indicated the textual references are insufficient to achieve the district’s multicultural education mathematics outcome. To further infuse the mathematics curriculum with the histories and contributions of persons of diverse racial, ethnic, and cultural backgrounds, specific multicultural connections can be found on DocuShare. This information can be used to transform the mathematics curriculum with the histories and contributions of diverse perspectives and can serve as an appropriate part of the teacher’s introduction of the listed mathematics topics.

Some resources that outline the contributions other cultures have made to the development of mathematics and teaching mathematics for multiple cultural perspectives are listed below:

Ascher, M. (1991). Ethnomathematics: A multicultural view of mathematical ideas. Belmont, CA: Wadsworth.

Baumgart, J. K., Deal, D. E., Hildebrandt, E. H. C., & Hallerberg, A. E. (Eds.). (1989). Historical topics for the mathematics classroom. Reston, VA: National Council of Teachers of Mathematics.

Boyer, C. B., & Merzbach, U. C. (1991). A history of mathematics (2nd ed.). New York: John Wiley and Sons, Inc.

Consortium for Mathematics and its Applications. (1992). Historical notes: Mathematics through the ages. Lexington, MA: COMAP.

Cooney, M. P. (Ed.). (1996). Celebrating women in mathematics and science. Reston, VA: National Council of Teachers of Mathematics.

Joseph, G. G. (1992). On the crest of the peacock: Non-European roots of mathematics. London: Penguin Books.

Nelson, D., Joseph, G. G., & Williams, J. (1993). Multicultural mathematics: Teaching mathematics from a global perspective. Oxford: Oxford University Press.

Secada, W. G., Fennema, E., & Adajian, L. B. (1995). New directions for equity in mathematics education. Cambridge: Cambridge University Press.

Zaslavsky, C. (1973). Africa counts. Boston: Prindle, Weber, & Schmidt.

**Where Can We Find Research to Support our SIP/PLC/Data Teams?**

Increasingly, education leaders look to research when making educational decisions. It is important to understand what research can and cannot do. As Hiebert (1999) and Marzano (2007) have discussed, teaching takes place in a complex environment, and research does not provide definitive answers to questions. Instead, recommendations based on research rest on probability estimates—that is, what is likely to improve student learning—and recommendations will change over time as new information emerges. However, we do know a good deal from the research about effective mathematics teaching and learning (Reed 2008). This guide cites relevant research whenever possible and the Math Step 4 Data Team Guide is your primary resource for identifying research-informed instructional practices. In addition, one of NCTM's strategic initiatives in the last half-decade has been to link research and practice. As part of this initiative, NCTM has published a number of resources that you can consult to find additional research to improve teaching and learning:

*Disrupting Tradition: Research and Practice Pathways in Mathematics Education* (Tate, King, and Anderson 2011)

*Teaching and Learning Mathematics: Translating Research for Secondary School Teachers* (Lobato and Lester 2010)

*Teaching and Learning Mathematics: Translating Research for Elementary School Teachers* (Lambdin and Lester 2010)

*Teaching and Learning Mathematics: Translating Research for School Administrators* (Charles and Lester 2010)

*Second Handbook of Research on Mathematics Teaching and Learning* (Lester 2007)

*A Research Companion to "Principles and Standards for School Mathematics"* (Kilpatrick, Martin, and Schifter 2003)

NCTM's Research Briefs and Clips, available at www.nctm.org/researchbriefs.aspx

**What Are Effective Strategies to Support Students Who Struggle?**

In general, students with difficulties in mathematics should receive instruction that emphasizes all aspects of mathematical proficiency, with the same teaching and the learning principles applying to all students, including those with special needs (Baroody 2011). In a recent summary of the research on specific instructional strategies that have consistently been found to be effective in teaching students who experience difficulties with mathematics, Gersten and Clarke (2007, p. 2) reached the following conclusions:

> For low-achieving students, the use of structured peer-assisted learning activities, along with systematic and explicit instruction and formative data furnished both to the teacher and to students, appears to be most important. For special education students, explicit, systematic instruction that involves extensive use of visual representations appears to be crucial. In many situations with special education students, it is often advantageous for students to be encouraged to think aloud while they work, perhaps by sharing their thinking with a peer. These approaches also seem to inhibit those students who try too quickly and impulsively to solve problems without devoting adequate attention to thinking about what mathematical concepts and principles are required for the solution. Instruction should ideally be in a small group of no more than six and (*a*) address skills that are necessary for the unit at hand, (*b*) be quite explicit and systematic, and (*c*) require the student to think aloud as she or he solves problems or uses graphic representation to work through problem solving options. Finally, it should balance work on basic whole-number or rational-number operations (depending on

grade level) with strategies for solving problems that are more complex. These criteria should be considered in evaluating intervention programs for working with these types of students.

Another recent review of the research on instruction for students with learning disabilities in mathematics has similarly found that a systematic and explicit approach to instruction is most effective (Gersten et al. 2009a). This approach is characterized by teacher modeling, followed by students' practicing with similar problems and receiving specific and immediate feedback from the teacher as they verbalize and explain their solutions and understandings, followed by ongoing cumulative review of key concepts (Clarke et al. 2011).

NCTM recently published a useful resource, *Achieving Fluency: Special Education and Mathematics* (Fennell 2011), which offers teachers and leaders additional specific strategies to support learners who struggle with mathematics.

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